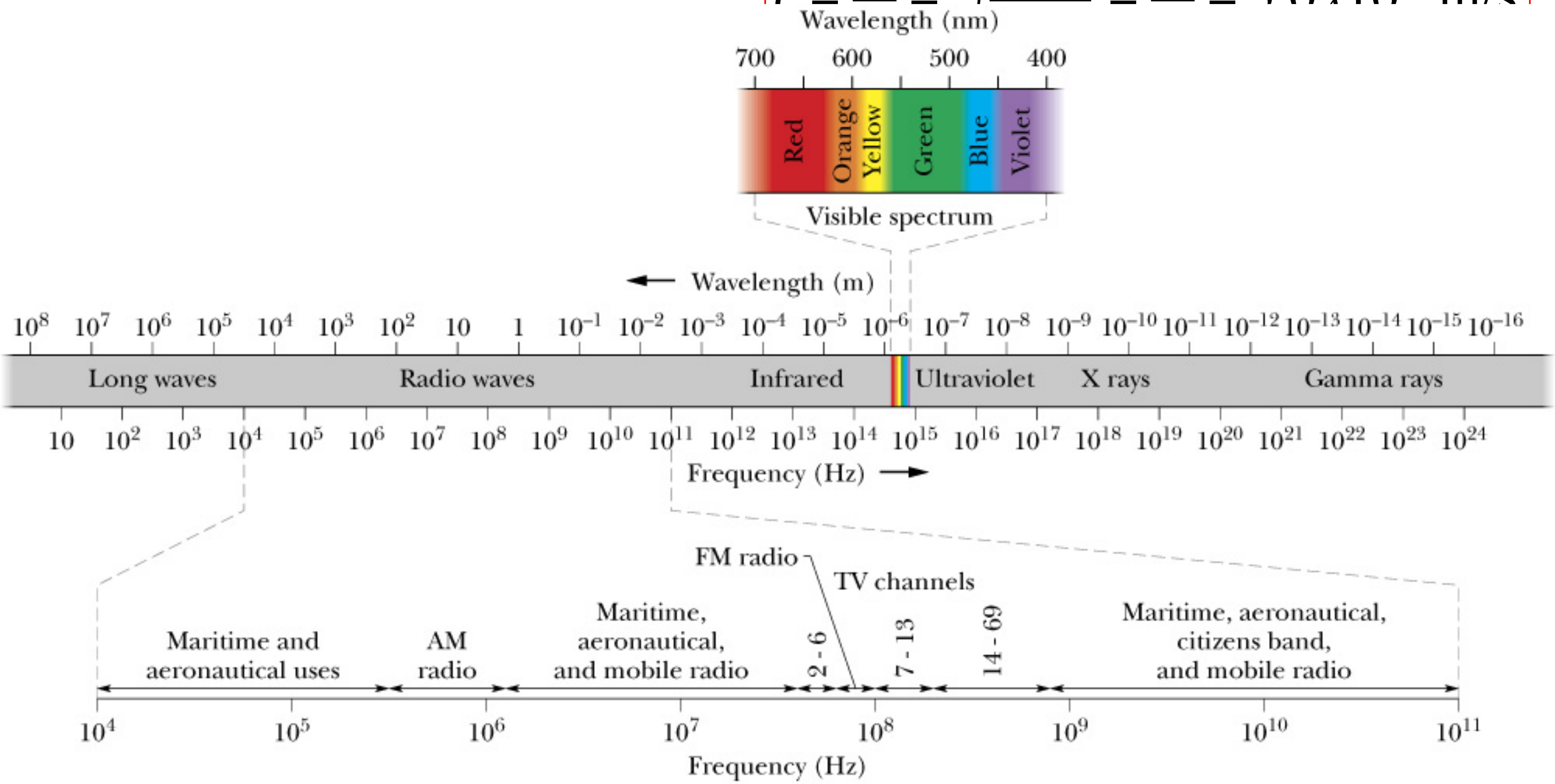
Properties of Light

- Light travels at speed

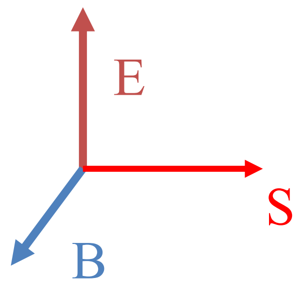
$$c = \frac{\omega}{\ } = \sqrt{\frac{1}{\ }} = \frac{E}{\ } = 3.0x10^8 \text{ m/s}$$

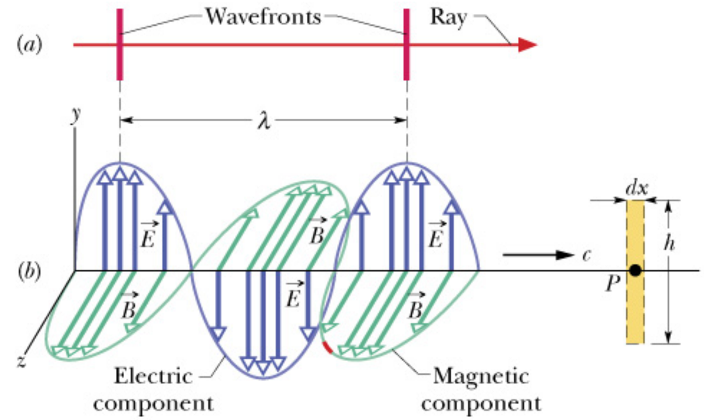


The Poynting Vector

The power transported by the wave and its direction is quantified by the **Poynting vector**.

$$\boxed{\mathbf{S} = \frac{1}{\mu_0}\mathbf{E}\times\mathbf{B}}$$





For a wave, since
E is perpendicular to B:

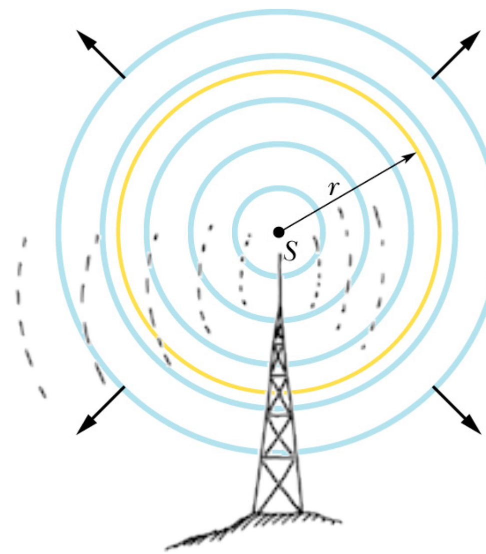
$$|S| = \frac{1}{\mu_0}EB = \frac{1}{c\mu_0}E^2$$

Units: Watt/m^2

Intensity is the ***average*** of the Poynting vector:

$$I = \bar{S} = \frac{1}{c\mu_0}\overline{E^2} = \frac{1}{c\mu_0}E_m{}^2\,\overline{\sin^2(kx-\omega t)}$$

$$I = \frac{1}{2c\mu_0}E_m{}^2 \quad \text{or,} \quad I = \frac{1}{c\mu_0}E_{rms}{}^2$$

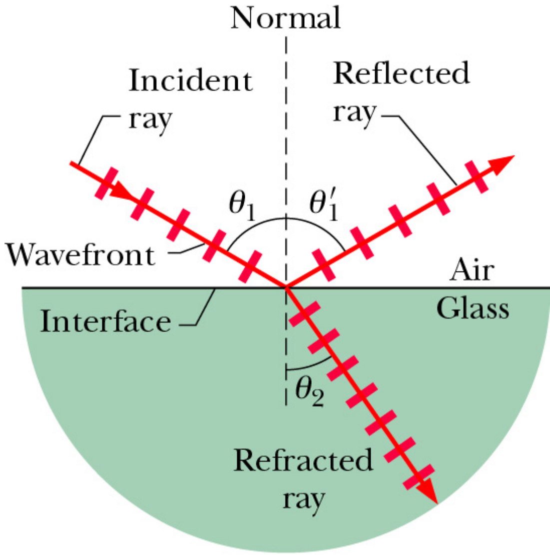


spherical waves

$$I = \frac{P_s}{4\pi r^2}$$

Reflection and Refraction

When light finds a surface separating two media (air and water, for example), a beam gets **reflected** and another gets **refracted** (transmitted).



(*b*)

Law of reflection: the angle of incidence θ_1 equals the angle of reflection θ'_1.

Law of refraction (Snell's law):

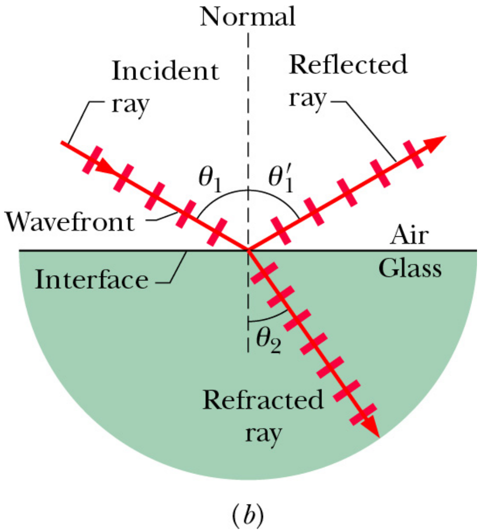
$$n_2 \sin\theta_2 = n_1 \sin\theta_1$$

n is the index of refraction of the medium.

In vacuum, n=1. In air, n~1. In all other media, n>1. $n = \frac{c}{v}$

Angle of total internal reflection $\Rightarrow \sin\theta_c = \frac{n_2}{n_1}$

Reflection and Refraction



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Plane Mirrors

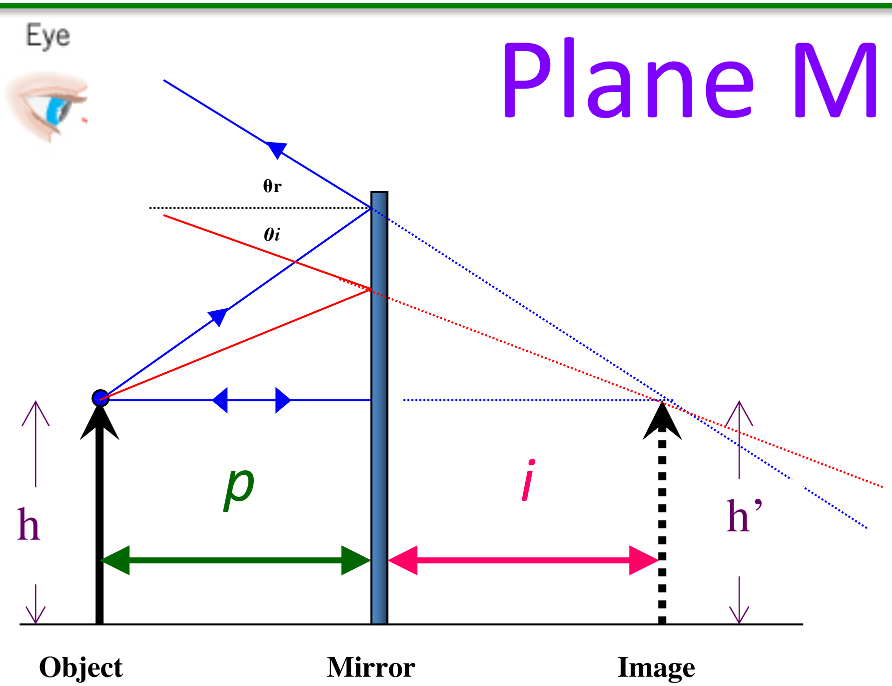
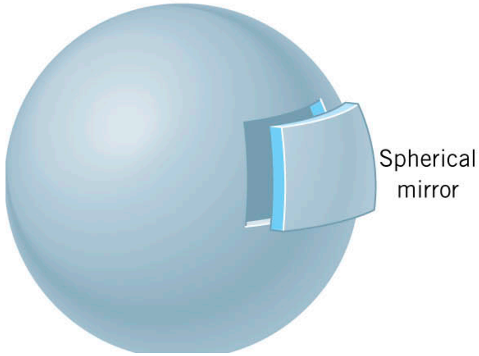


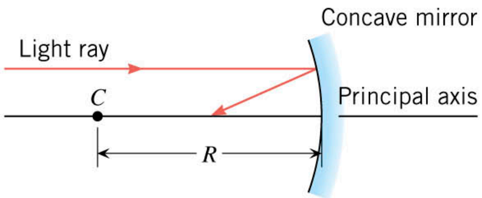
Image and Object distances: i and p

For Plane Mirror: $\boxed{i = -p}$

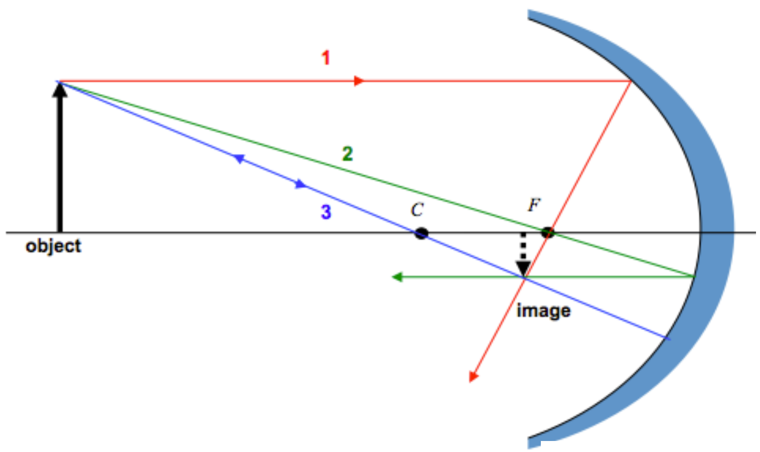
Virtual, since the light rays only appear to come from the image, but they do not actually emanate from there.

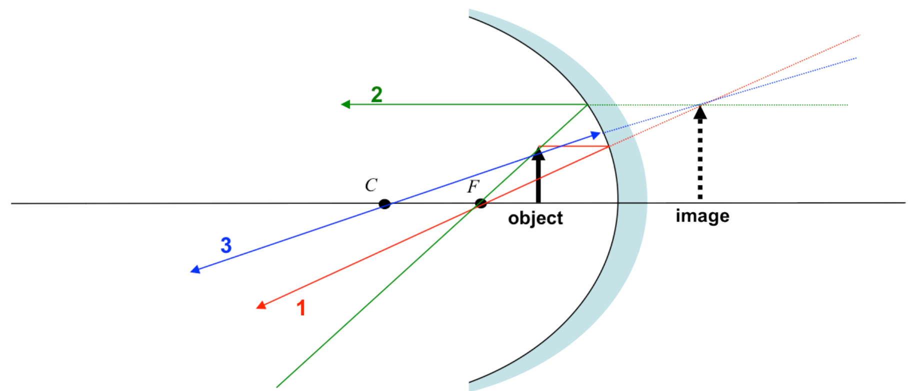
Spherical Mirrors



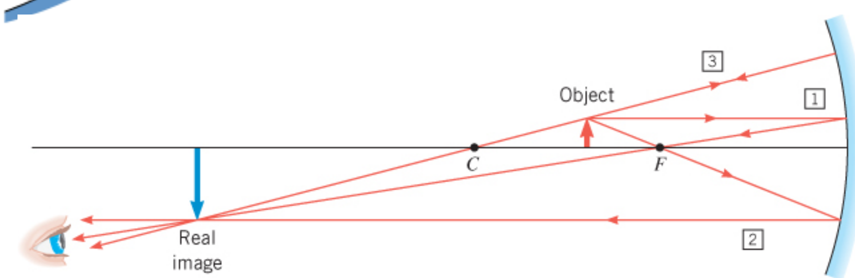


1. A paraxial ray from the object. It passes thru the focal point F upon reflection.
2. A ray from the object that passes thru F and then strikes the mirror. Upon reflection it leaves parallel to the principle axis.
3. A ray from the object that passes thru C and then strikes the mirror. Upon reflection it retraces its original path.

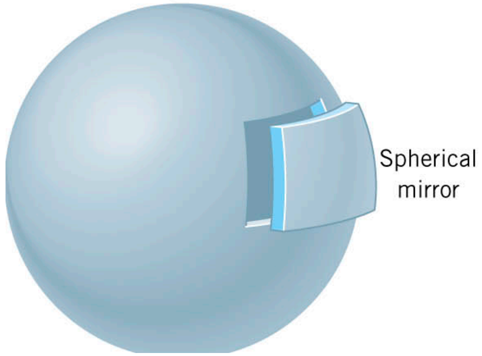


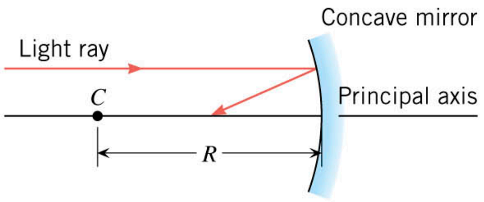


We see that the reflected rays don't intersect.



Spherical Mirrors





1. A paraxial ray from the object. It passes thru the focal point F upon reflection.
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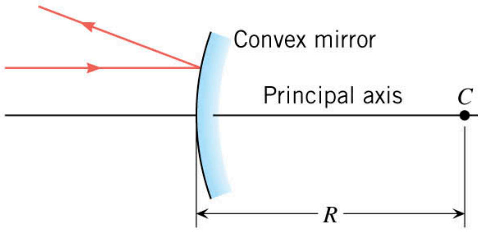
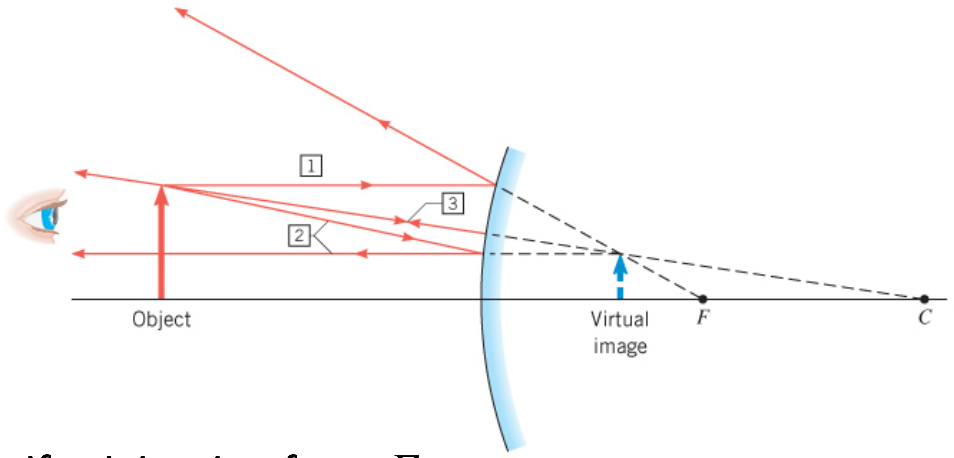
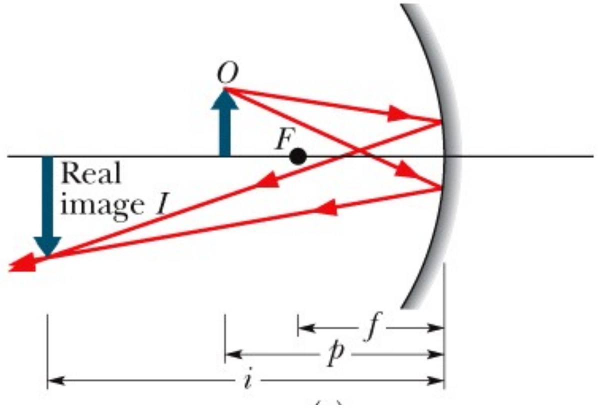
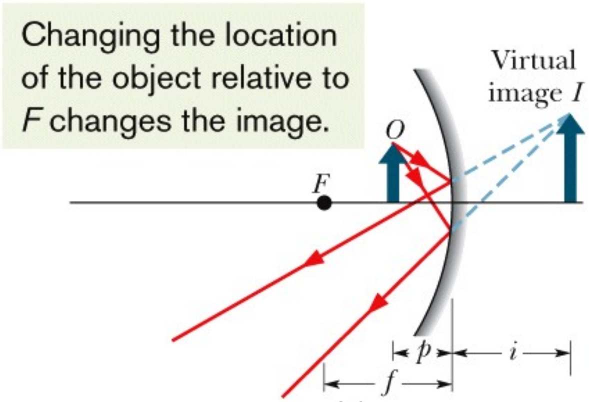
3. A ray from the object that passes thru C and then strikes the mirror. Upon reflection it retraces its original path.





1. A paraxial ray from the object reflects as if originating from F.
2. A ray from the object that heads toward F gets reflected parallel to the principle axis.
3. A ray from the object that heads toward C gets reflected back along the same path as if originating from C.





Sign Conventions

$$\frac{1}{p}+\frac{1}{i}=\frac{1}{f}$$

Focal Length:

f is positive for concave mirrors
f is negative for convex mirrors

Object Distance

p is positive if the object is in front of the mirror
~~p is negative if the object is behind the mirror~~

Image Distance

i is positive if the image is in front of the mirror- **real image**
i is negative if the image is behind the mirror- **virtual image**

$$m=\frac{\text{Image height}}{\text{Object height}}=\frac{h'}{h}=-\frac{i}{p}$$

If m > 1, the image is enlarged.
If m < 1, the image is reduced.
If m is positive, the image is upright.
If m is negative, the image is inverted.

Formation of Images by Lenses

(NOTE: f comes from thin lens formula)

Mirrors and lenses both form images:

- mirrors reflect light to produce image
- light *passes through* lenses to create image

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \quad \text{(thin lens in air)},$$

How do we determine what image is produced?

- ray tracing diagrams: determine location, size, orientation of image
- assume lens is thin relative to focal length
- **i** and **p** are both measured from center of lens
- assume object is located on principal axis to *left* of lens (unless otherwise stated)

Again, we will use three principle rays to locate the image:

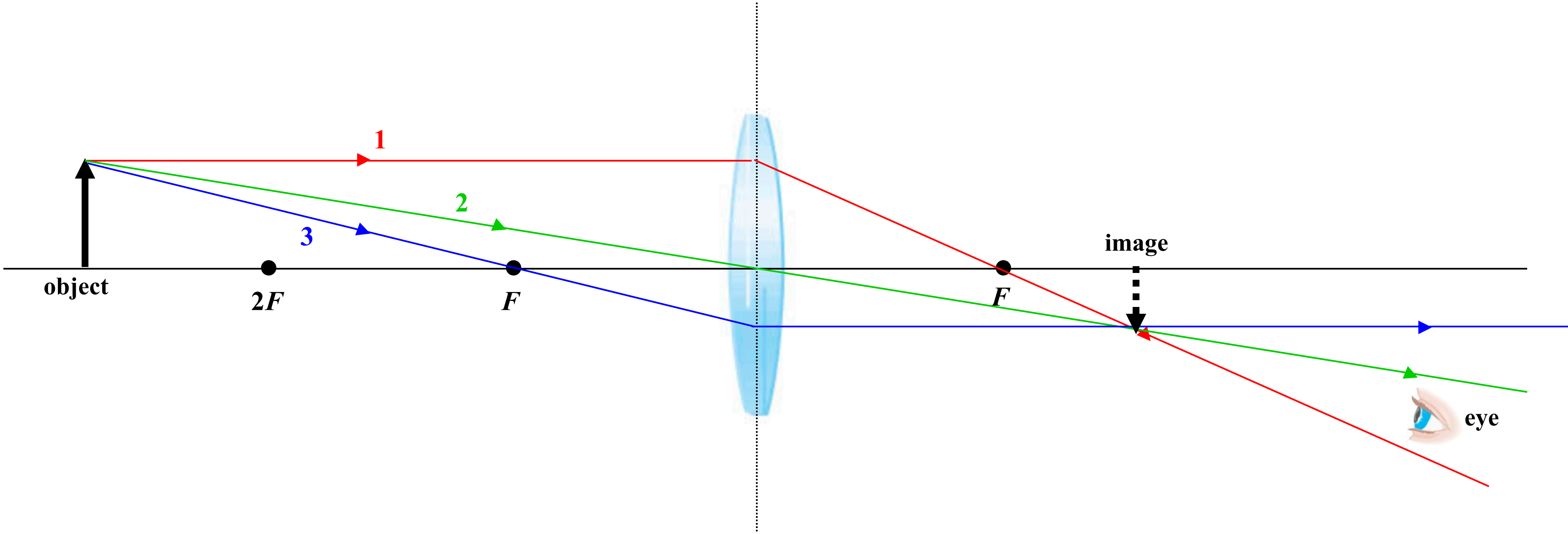
1. A ray traveling parallel to the principal axis gets refracted thru the focal point.

2. A ray traveling thru the center of the lens is unaffected.

3. A ray traveling thru the focal point gets refracted parallel to the principal axis.

There are 3 cases for a converging lens.

Case 1: The object is located beyond 2*F*:



Ray 1 **is a paraxial ray from the object that gets refracted thru *F*.**

Ray 2 **travels thru the center of the lens unaffected.**

Ray 3 **travels thru *F* and comes out parallel.**

Now we can see where the rays intersect, and thus the image position.

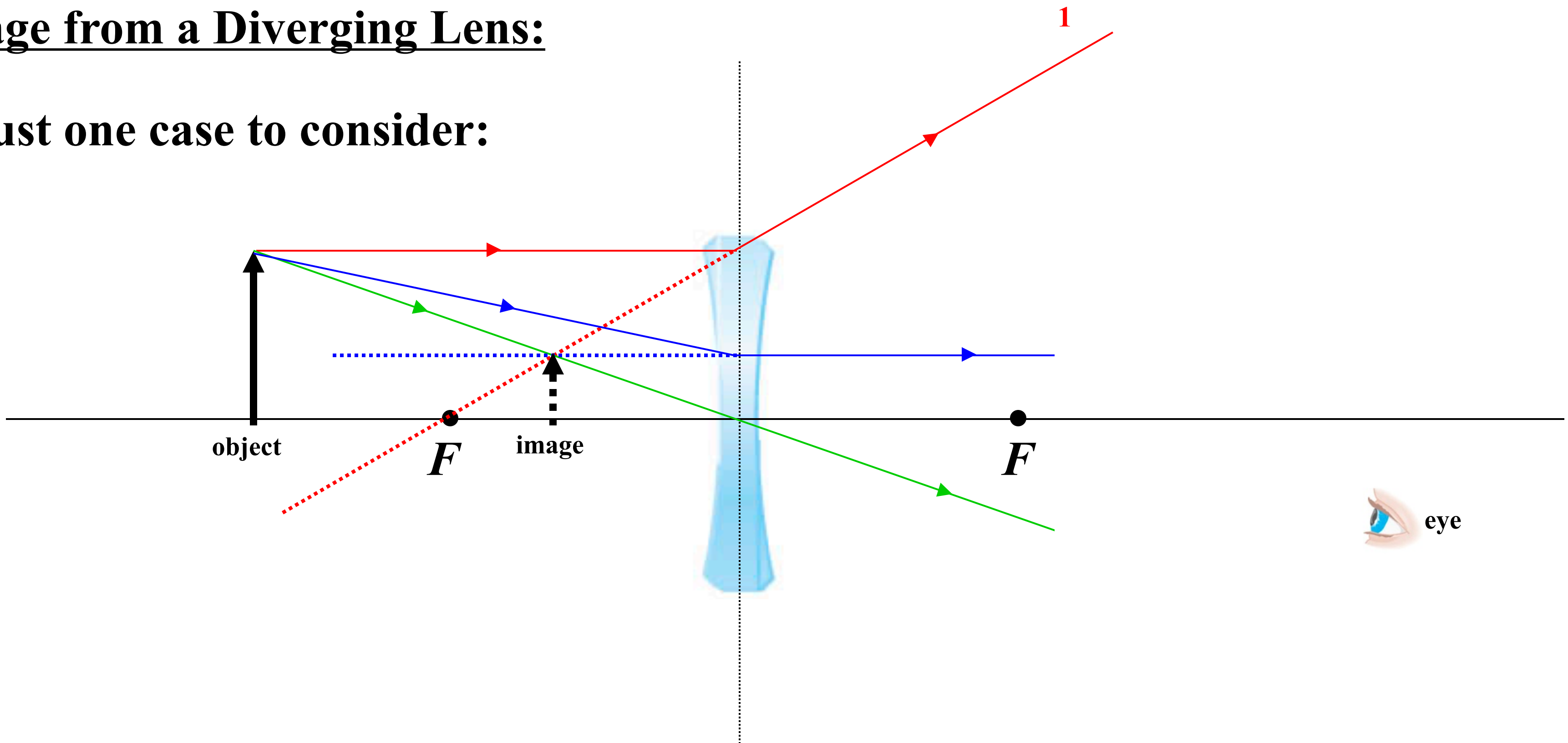
What are the image properties?

Real, inverted, and reduced.

Camera

Image from a Diverging Lens:

Just one case to consider:



Ray 1 is a paraxial ray from the object that gets refracted <u>as if it came from *F*.</u>

Ray 2 travels thru the center of the lens unaffected.

Ray 3 travels toward the opposite *F* and comes out parallel.

Now we can see where the rays intersect, and thus the image position.

<u>What are the image properties?</u> **<u>Virtual, upright, and reduced.</u>**

34.4 The Thin Lens and Magnification Equation

We used the law of reflection to find a relationship between the focal length (f), the object distance (p), and the image distance (i) for mirrors.

We can do the same thing for lenses using Snell's law.

The result we find is the same:

$$\boxed{\frac{1}{f} = \frac{1}{p} + \frac{1}{i}}$$

Thin Lens Equation

As before, the magnification is:

$$\boxed{m = \frac{h_i}{h_o} = \frac{-i}{p}}$$

Sign Conventions: Lenses

This assumes the <u>object is located to the left of the lens</u>, and your eye is located to the right of the lens.

<u>Focal length:</u>

f is positive for converging (convex) lenses.

f is negative for diverging (concave) lenses.

<u>Object Distance:</u>

p is positive for objects left of the lens (real). *[from which light diverges]*

p is negative for objects right of the lens (virtual). *[towards which light converges]**

<u>Image Distance:</u>

i is positive for images right of the lens (real).

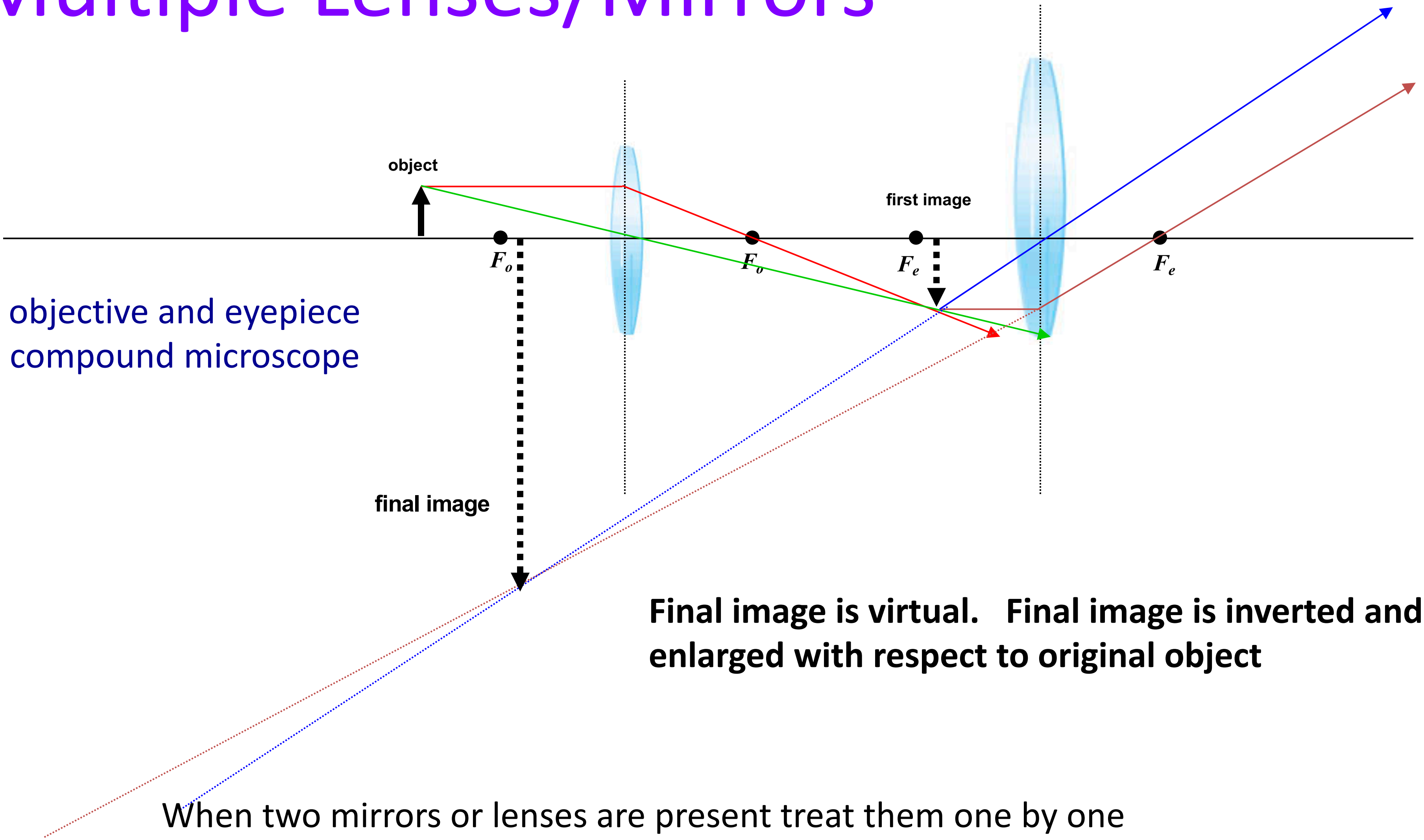
i is negative for images left of the lens (virtual).

<u>Magnification:</u>

m is positive for images upright wrt the object.

m is negative for images inverted wrt the object.

Multiple Lenses/Mirrors



The objective and eyepiece of a compound microscope

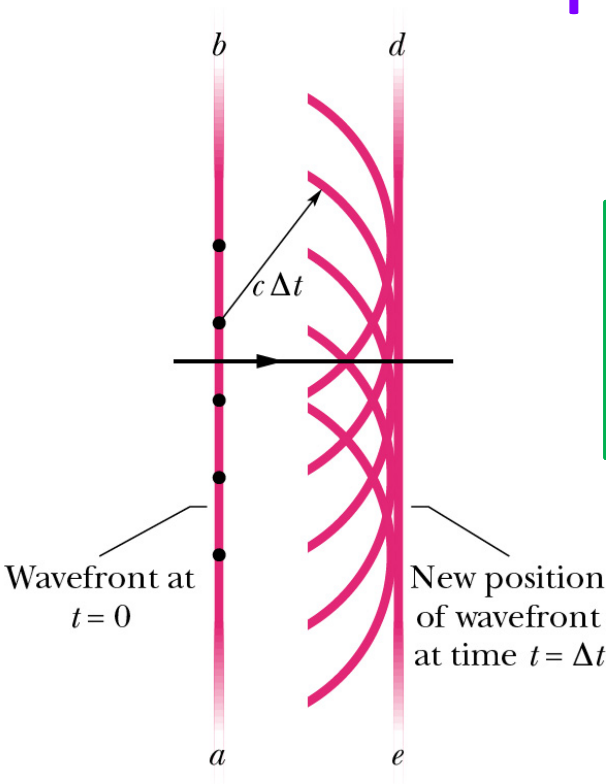
Final image is virtual. Final image is inverted and enlarged with respect to original object

When two mirrors or lenses are present treat them one by one

1. Draw/calculate location of image from first lens/mirror only
2. Take image from 1st component to be object of 2nd lens/mirror
3. Draw final image
4. Overall magnification is product of two m's $M = m_1 m_2$

Huygen's Principle:

Light *is* a wave



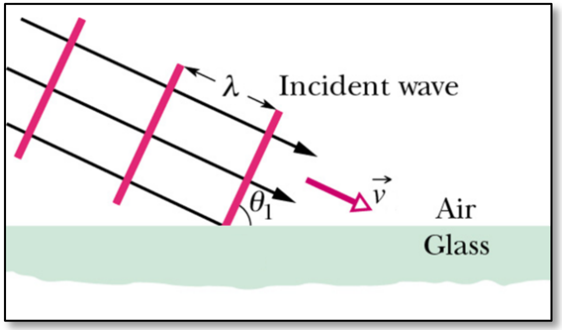
All points in a wavefront serve as point sources of spherical secondary wavelets.

Christian Huygens
1629-1695

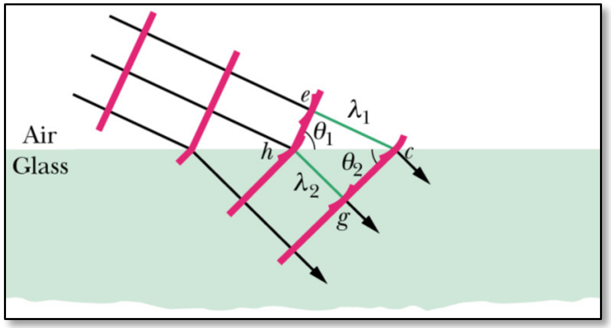
Method:

- Assign points along a wavefront
- Let the spherical waves expand for a time
- The new wavefront is the tangent line

Reflection



Refraction



Snell's Law!!!

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1}$$

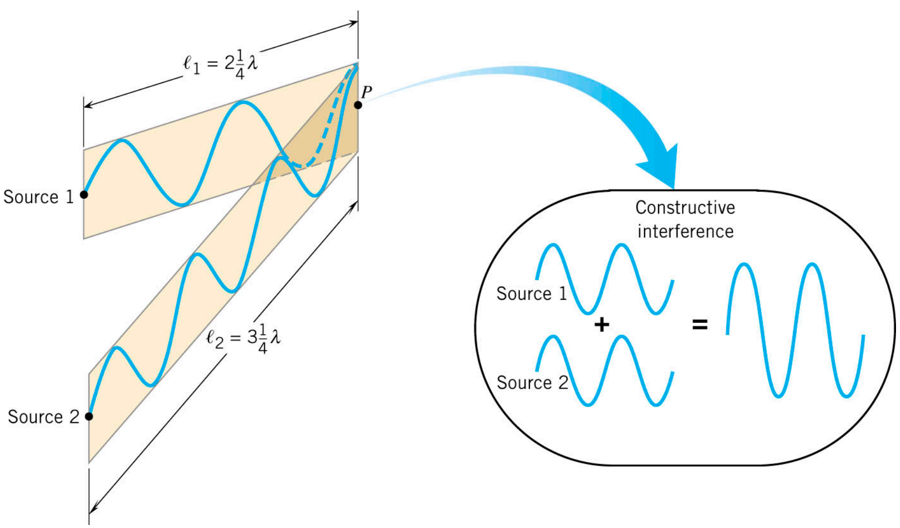
$$\boxed{\frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2} = \frac{n_2}{n_1}}$$

Review: Interference

Take two waves of equal amplitude and wavelength
and have them meet at a common point:

Define Optical Path Difference (ΔL = OPD):

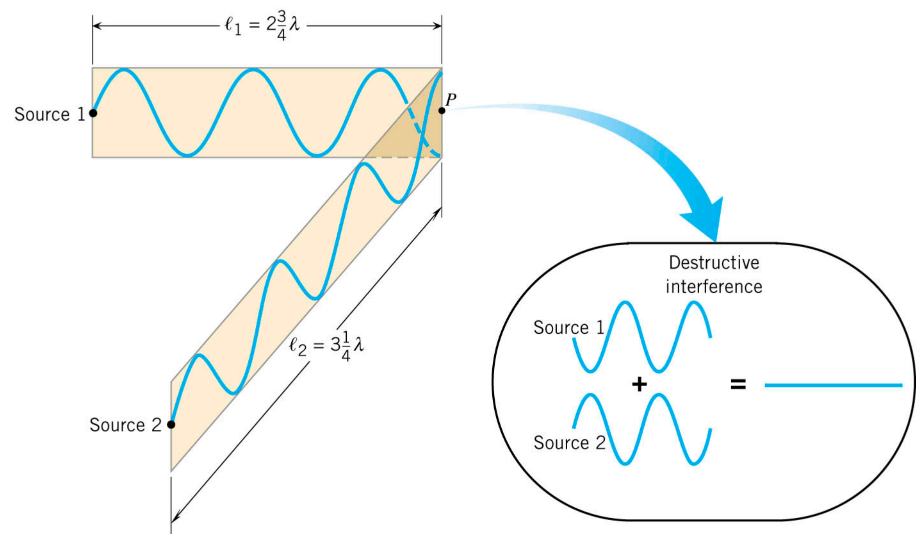
OPD = The difference in distance that two waves travel.



If the two waves are **in-phase**, then they meet crest-to-crest and trough-to-trough.

→ **Constructive Interference (CI).**

$$\Delta L = \mathrm{OPD} = \mathrm{m}\lambda,\ \mathrm{m} = 0,\ 1,\ 2,\ \ldots$$



If the two waves are **out-of--phase**, then they meet crest-to-trough and amplitudes cancel out

→ **Destructive Interference (DI).**

$$\Delta L = \mathrm{OPD} = (\mathrm{m} + \tfrac{1}{2})\lambda,\ \mathrm{m} = 0,\ 1,\ 2,\ \ldots$$

Phase Differences

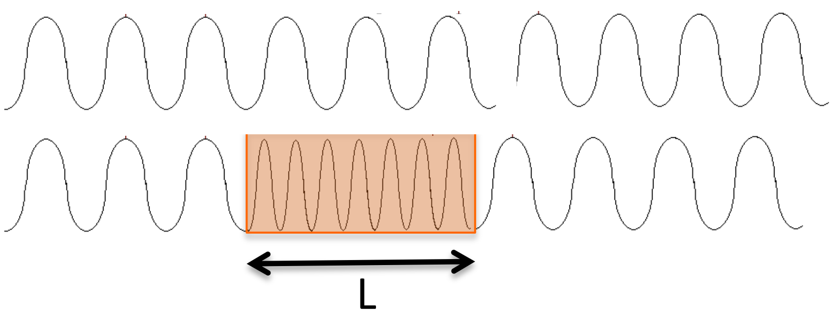
Differences in path lengths
→ interference phase differences

$$\Delta L = m\lambda \quad m = 0,1,2\ldots \quad \text{(CI)}$$

$$\Delta L = \left(m + \frac{1}{2}\right)\lambda \quad m = 0,1,2\ldots \quad \text{(DI)}$$

Another way to think of this??
→ by traveling different path lengths, waves go through a different number of wavelengths

Is there another way to travel a different number of wavelengths???



$$\frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2} = \frac{n_2}{n_1}$$

Number of wavelengths traveled in media *n*:

$$N_n = \frac{L}{\lambda_n} = \frac{L}{\left(\lambda / n_n\right)}$$

Difference in # of wavelengths over L for to initially in phase waves:

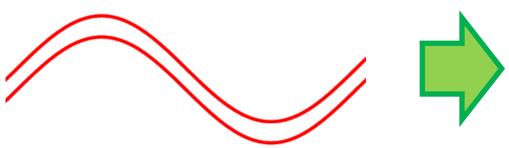
$$N_1 - N_2 = \frac{L}{\lambda}(n_1 - n_2)$$

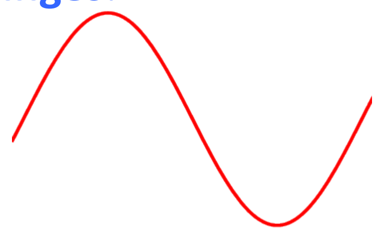
Young’s Double Split Experiment

Interference Fringes

Constructive interference fringes:

Waves arrive *in phase*…



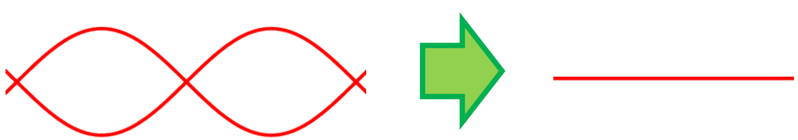


… giving a **bright spot**.

$$\boxed{d\sin\theta = m\lambda}$$

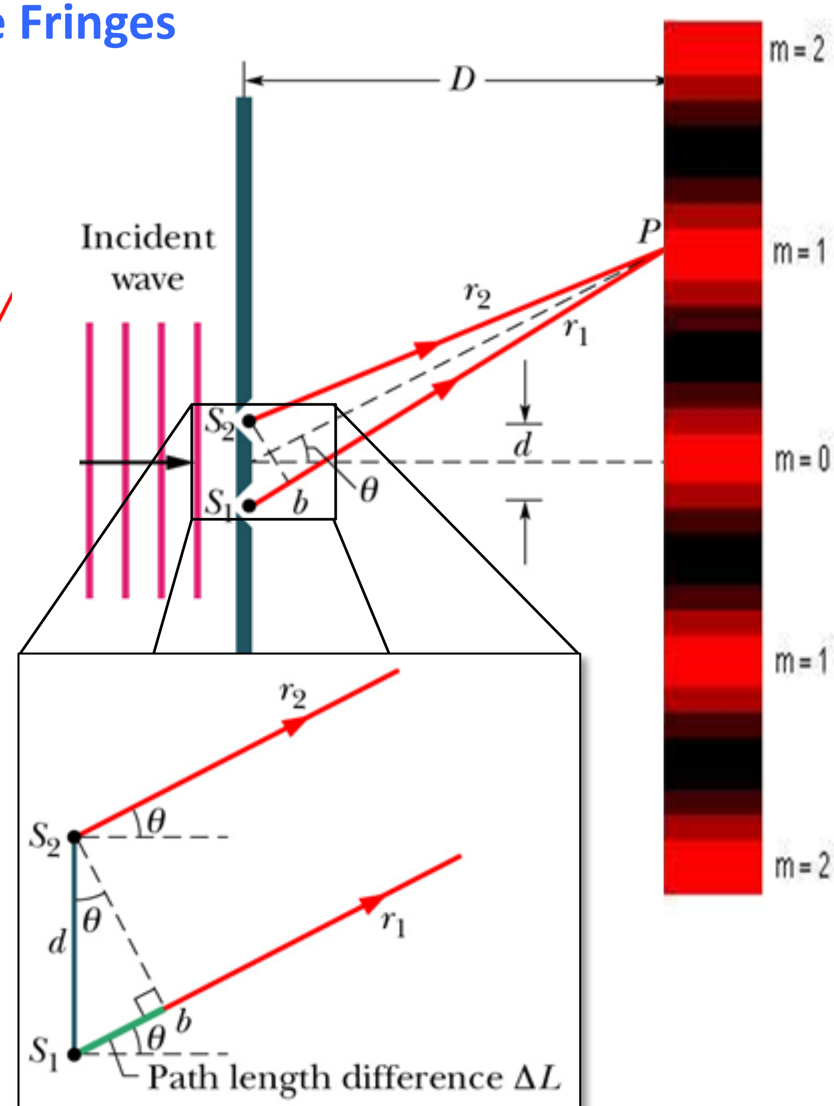
Destructive interference fringes:

Waves arrive *out of phase*…



… giving a **dark spot**.

$$\boxed{d\sin\theta = (m+\tfrac{1}{2})\lambda}$$



m is the **order** of the fringe.

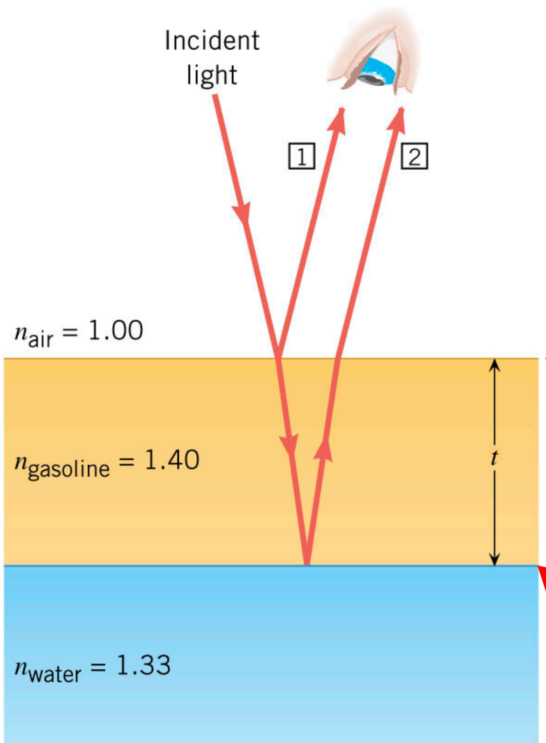
Thin Films

For thin films then the following is used:

$$\boxed{\Delta l + (\text{any phase shifts}) = \text{Interference condition}}$$

$$2t + (\text{phase shifts}) = \begin{cases} m\lambda, & \textbf{Film appears bright} \\ (m+\tfrac{1}{2})\lambda, & \textbf{Film appears dark} \end{cases}$$

Wavelength changes based on *n*
→ what wavelength do we choose?



Smaller *n* → larger *n*
→ Phase shift (½ λ)!

Larger *n* → smaller *n*
→ No phase shift!

The optical path difference occurs inside the film, so the index of refraction that is important here is n_{film}.

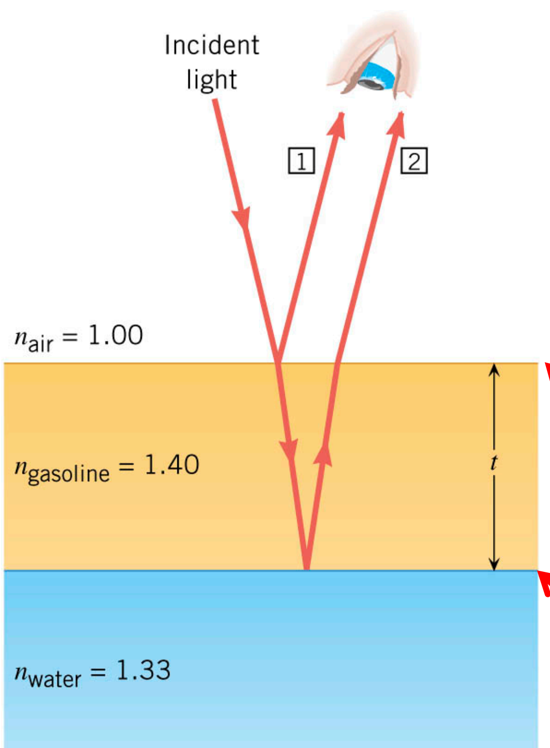
$$\lambda_{film} = \frac{\lambda_{vac}}{n_{film}}$$

$$2t + (\text{phase shifts}) = \begin{cases} m\lambda_{film} \\ (m+\frac{1}{2})\lambda_{film} \end{cases} = \begin{cases} m\dfrac{\lambda_{vac}}{n_{film}}, & \textbf{Film appears bright} \\ (m+\frac{1}{2})\dfrac{\lambda_{vac}}{n_{film}}, & \textbf{Film appears dark} \end{cases}$$

For Thin-Film Interference in Air we will always have: $n_{medium} > n_{air}$

A phase shift of $\frac{1}{2}\lambda$

at reflection from air-medium interface



Smaller n → larger n

→ Phase shift (½ λ)!

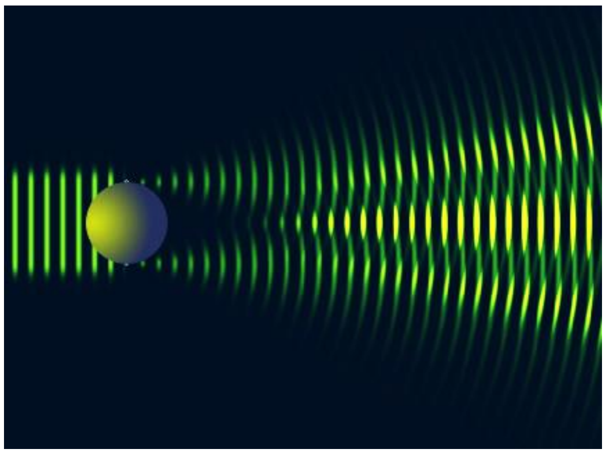
Larger n → smaller n

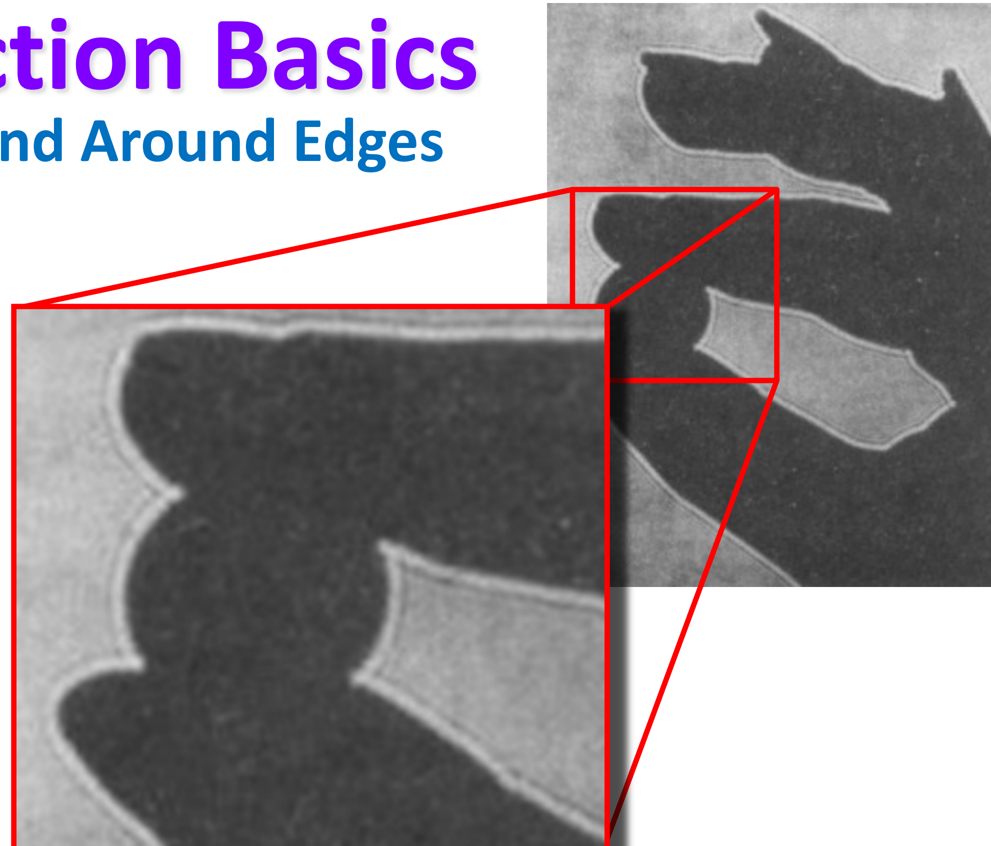
→ No phase shift!

Diffraction Basics

Waves Bend Around Edges

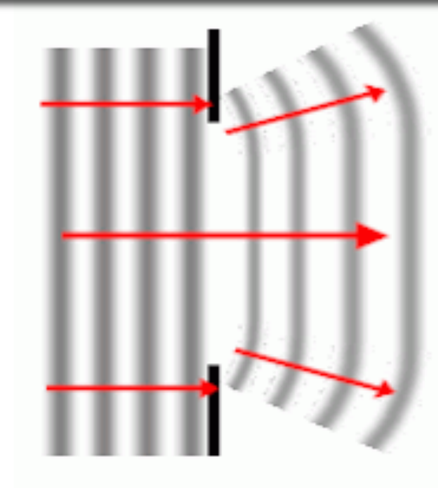
Diffraction occurs whenever light encounters the edge of an object. Light will diffract around anything.

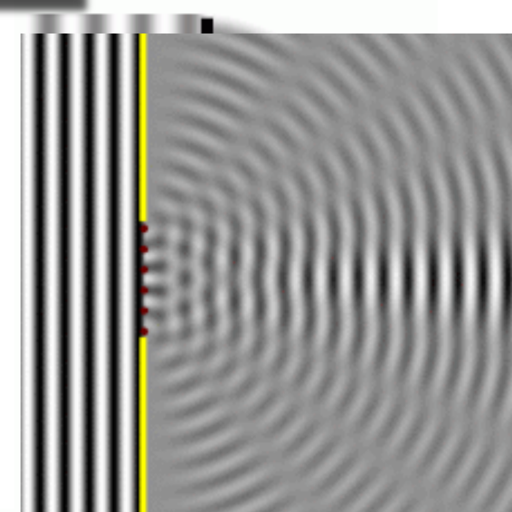




Diffraction is easiest to see when …

(a)the light is coherent
(b)the edge of the object is sharp
(c)the object is about as large as the wavelength of light.





Lasers have made it easy to create diffraction patterns today.

I. Single Slit Diffraction

- Conclusion is that we get minima at all

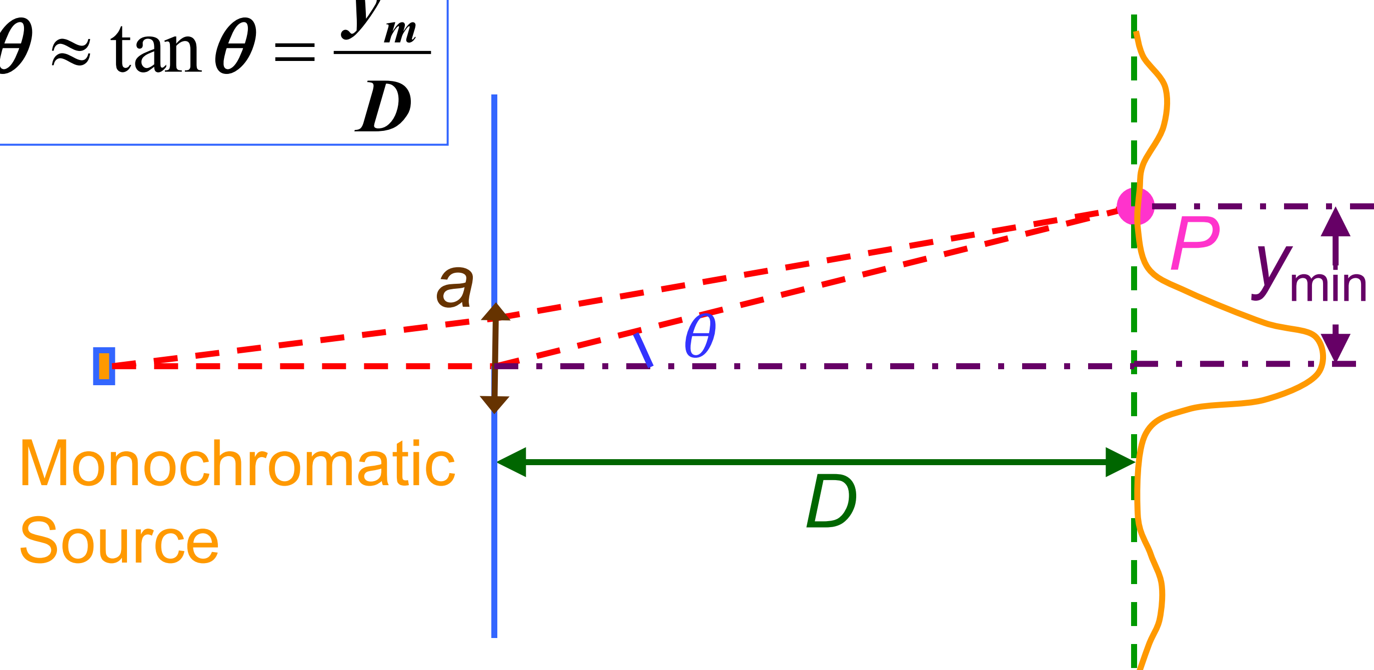
$$\boxed{a\sin\theta_{\min} = m\lambda}$$

- We can play the same trick with small angle formulas that we played in the two slit problem to write

$$\boxed{\sin\boldsymbol{\theta} \approx \tan\boldsymbol{\theta} = \frac{\boldsymbol{y_m}}{\boldsymbol{D}}}$$

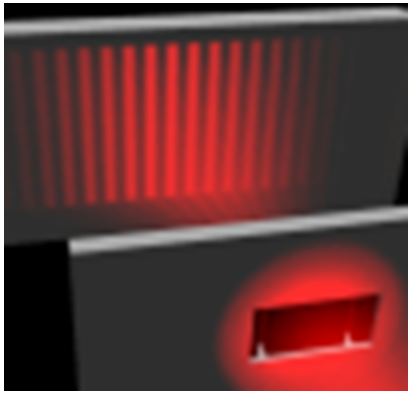
- So that

$$\boxed{y_{\min} = \frac{m\lambda D}{a}}$$

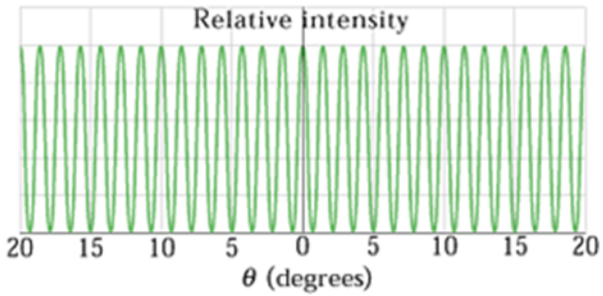


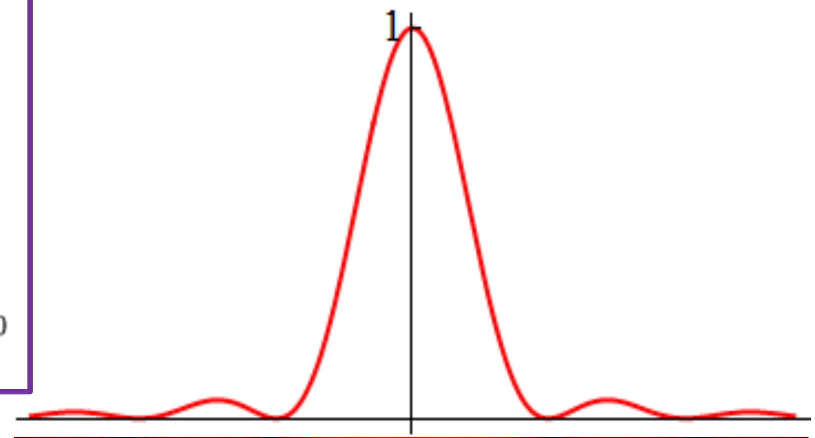
Diffraction by a Double Slit

Combining the Two Results



Double Slit Interference



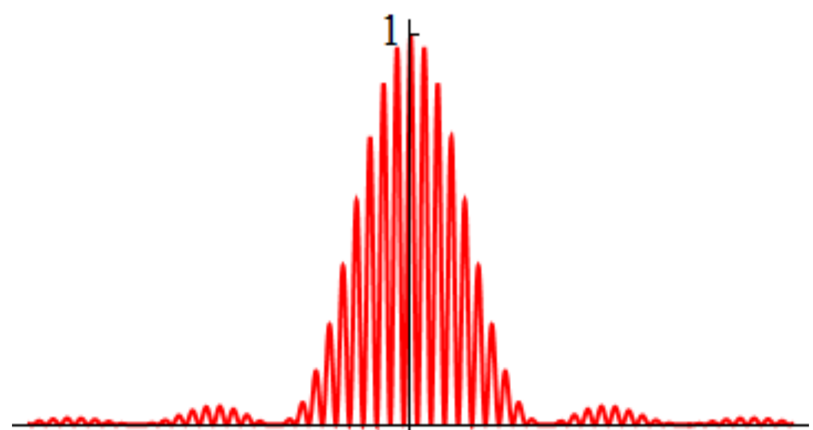


Single Slit Diffraction:

Diffraction & Interference:

Intensity on Screen

$$\frac{I}{I_m} = \cos^2(\beta)\,\frac{\sin^2\alpha}{\alpha^2}$$

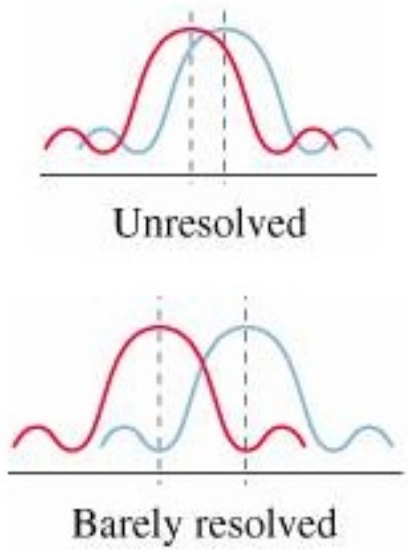


Diffraction due to slit width: $\alpha = \dfrac{\pi a}{\lambda}\sin\theta$

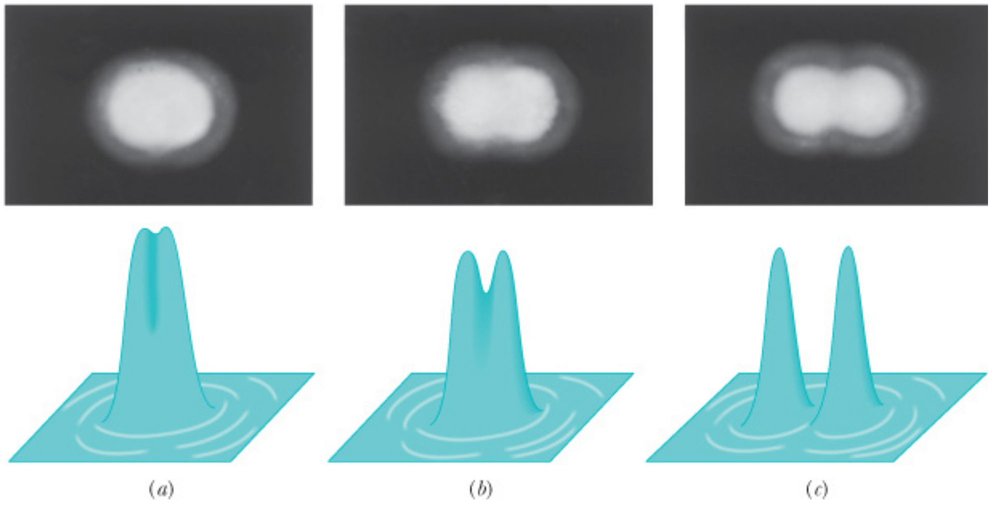
Interference due to separation: $\beta = \dfrac{\pi d}{\lambda}\sin\theta$

Resolvabilty

The Raleigh Criterion



Rayleigh came up with a rule of thumb for deciding when two objects can be resolved. He chose to call two sources **resolvable** (*i.e.*, distinguishable) if their **images** were separated by a distance equal to the distance to the first diffraction minimum.



d = diameter of circular aperture

The first-order minimum:

$$\sin\theta = 1.22\frac{\lambda}{d}$$

Minimal angle for resolution:

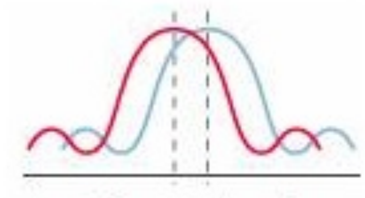
$$\theta_R = \sin^{-1}\left(1.22\frac{\lambda}{d}\right)$$

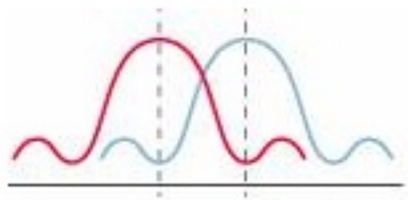
This angle is very small, hence

$$\boxed{\theta_R = 1.22\frac{\lambda}{d}}$$

Resolvabilty

The Raleigh Criterion





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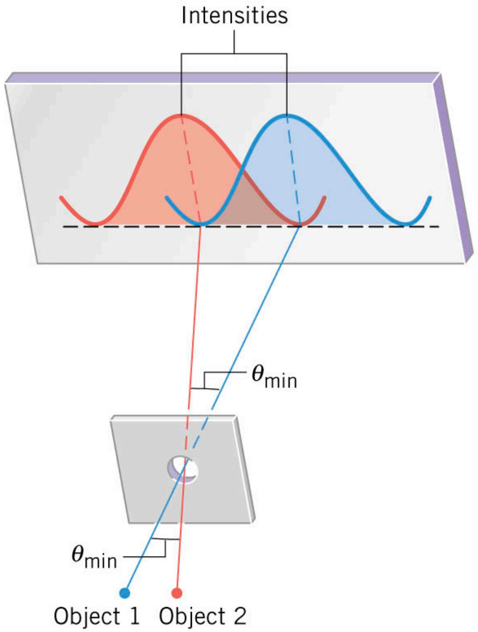
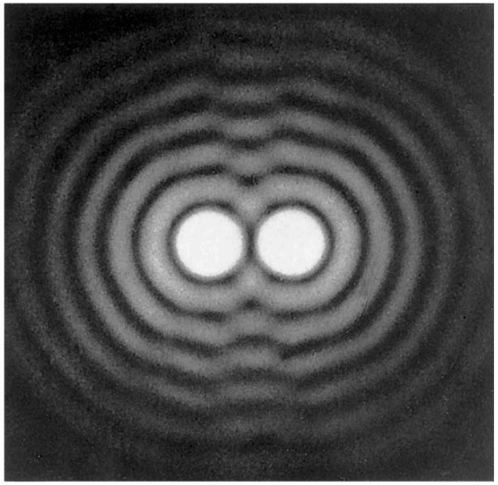


(a)



(b)

The first-order minimum:

$$\sin\theta = 1.22\frac{\lambda}{d}$$

Minimal angle for resolution:

$$\theta_R = \sin^{-1}\left(1.22\frac{\lambda}{d}\right)$$

This angle is very small, hence

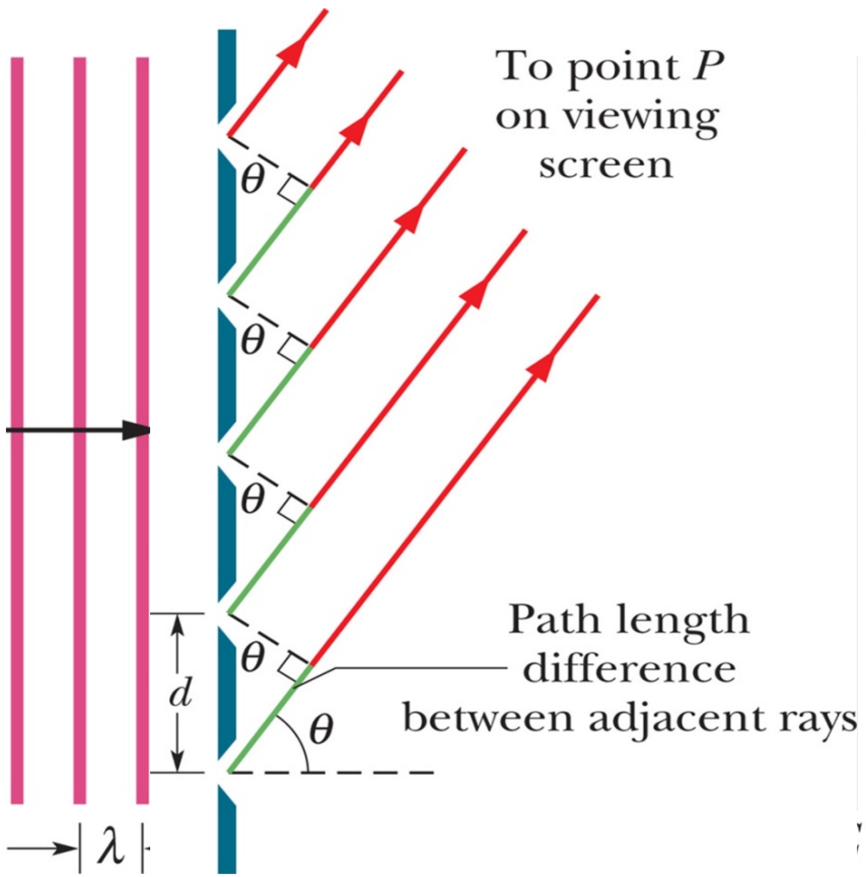
$$\boxed{\theta_R = 1.22\frac{\lambda}{d}}$$

Diffraction Gratings

Multiple Slit Diffraction and Interference

A device with N slits (rulings) can be used to separate different wavelengths of light that are contained in a single beam. How does it do this?

First, *how does a diffraction grating affect monochromatic light*?



Intensity Maxima

$$d \sin\theta = m\,\lambda$$

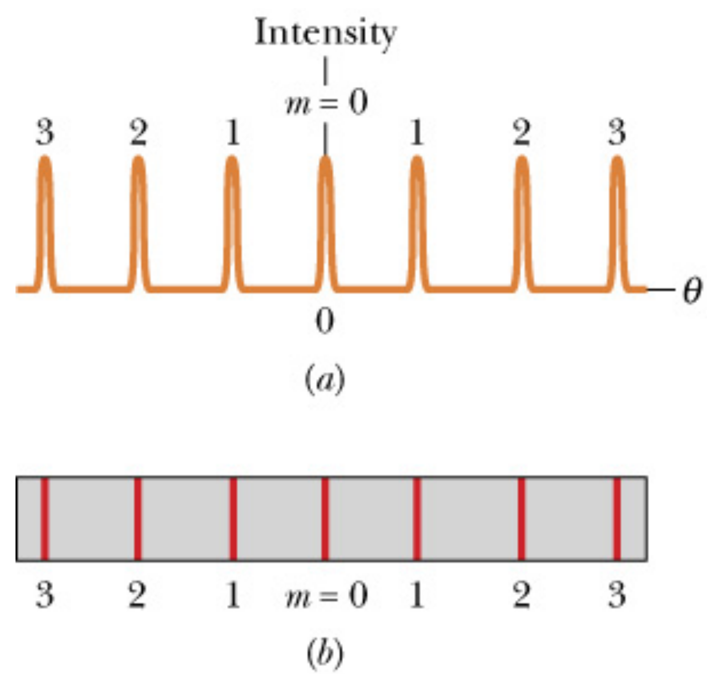


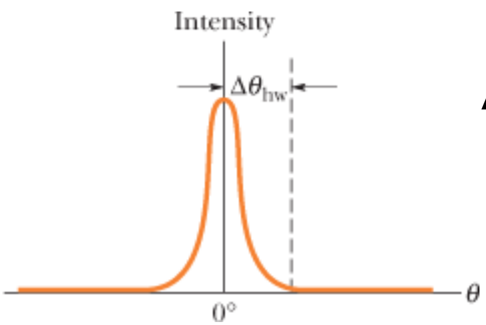
Fig. 36-18

Diffraction Gratings

Resolving a Particular Wavelength

The ability of the diffraction grating to resolve (separate) different colors of light depends on:

1. the width of the lines (maxima):



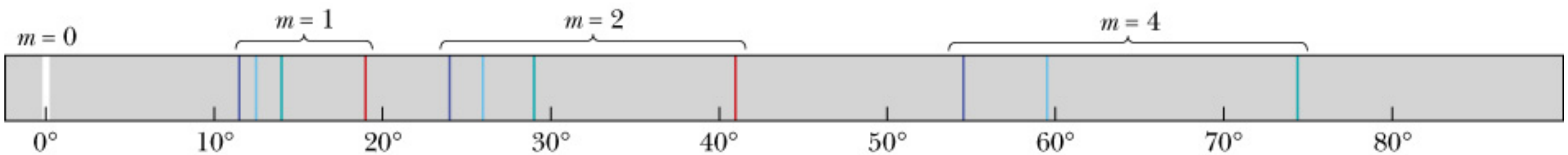
$$\Delta\theta_{hw} = \frac{\lambda}{Nd\cos\theta} \Rightarrow \Delta\theta_{hw} = \frac{\lambda}{Nd}$$

for central maxima (m = 0)

2. the dispersion D (how far apart are lines with different wavelengths):

$$\boxed{D = \frac{\Delta\theta}{\Delta\lambda}} \qquad \boxed{D = \frac{m}{d\cos\theta}}$$

$$\boxed{\text{Gives resolving power:} \quad R = \frac{\lambda_{avg}}{\Delta\lambda} = Nm}$$



To separate colors really well, we typically need <u>many</u> closely spaced lines!