22.4-22.6

#7: Distributions of Charge

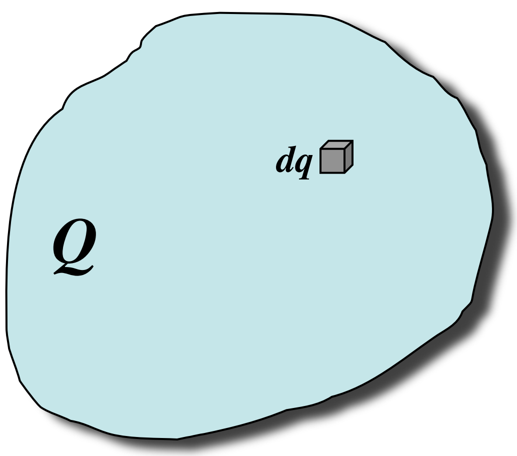
We often charge a macroscopic object.

Distributions of charges in the real world are usually complex and not well-known.

The electric field can be experimentally mapped by measuring the force on a known test charge.

We'll work not with point charges, but the ***charge density,*** a continuous quantity (ala Franklin's fluid).

Integration adds up contribution from the distribution of charge (typically nasty integral calculus in 3 dimensions).

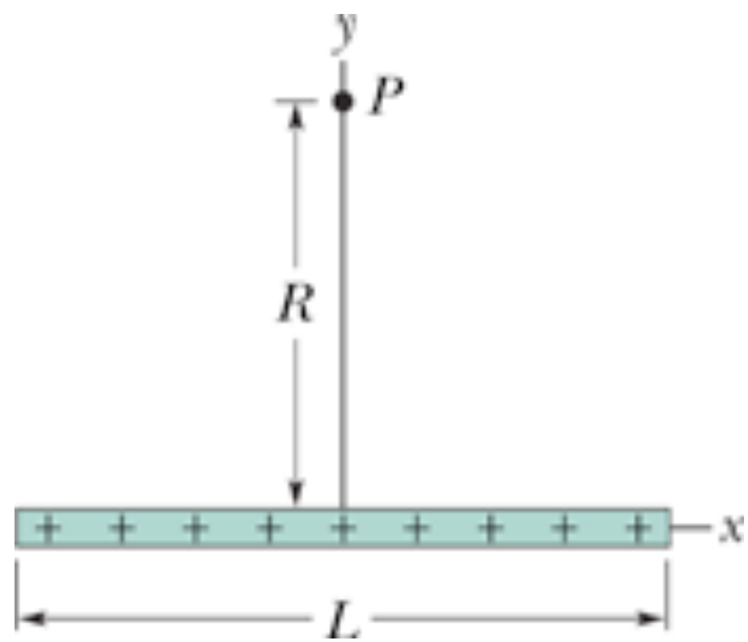


Name	Symbol	SI Unit		
Charge	q	C		
Linear charge density	λ	C/m	dq = λ ds	$Q = \int \lambda \cdot ds$
Surface charge density	σ	C/m^2	dq = σ dA	$Q = \int \sigma \cdot dA$
Volume charge density	ρ	C/m^3	dq = ρ dV	$Q = \int \rho \cdot dV$

$$\cancel{E = \int dE = \int \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}}$$

A positive charge of ***8 pC*** is spread uniformly along a thin nonconducting rod of length ***L = 16 cm.***

What are the (a) magnitude and (b) direction (relative to the positive direction of the x axis) of the electric field produced at point P, at distance ***R = 6 cm*** from the rod along its perpendicular bisector?



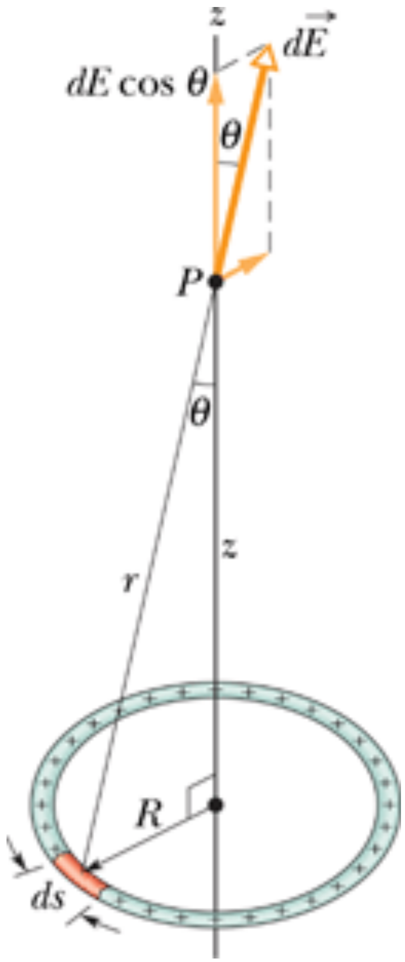
The Electric Hula Hoop

Consider points that lie along the z axis through the middle of the hoop

What direction is the net Electric Field going to point?

Along the z axis!

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{\lambda \cdot ds}{\left(z^2 + R^2\right)}$$



We need only add up components along the z axis, as the components in the xy plane will cancel

$$dE_z = dE\cos\theta = \frac{1}{4\pi\varepsilon_0}\frac{\lambda \cdot ds}{\left(z^2 + R^2\right)}\left(\frac{z}{\sqrt{z^2 + R^2}}\right)$$

$$E = \int \frac{1}{4\pi\varepsilon_0}\frac{\lambda}{\left(z^2 + R^2\right)}\left(\frac{z}{\sqrt{z^2 + R^2}}\right)ds$$

$$E = \frac{1}{4\pi\varepsilon_0}\left(z^2 + R^2\right)^{-3/2} z\lambda\left(2\pi R\right)$$

$$E = \frac{Qz}{4\pi\varepsilon_0}\left(z^2 + R^2\right)^{-3/2}$$

The Electric Hula Hoop

Consider points that lie along the z axis through the middle of the hoop

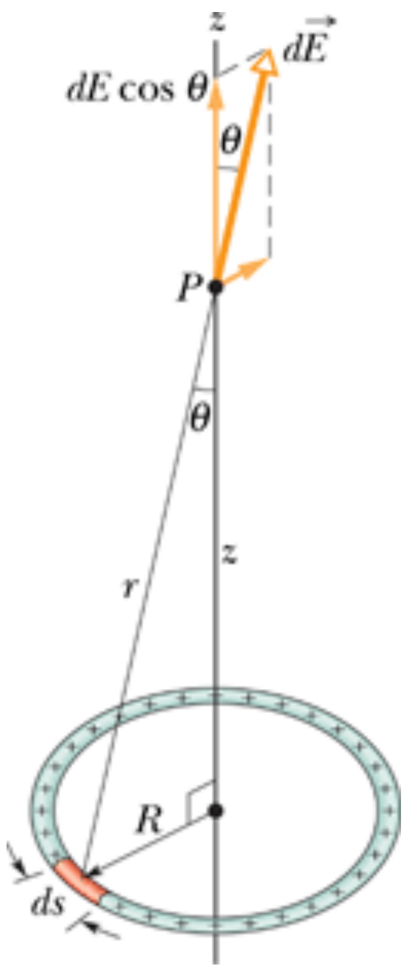
What direction is the net Electric Field going to point?

Check your limits

$$z >> R \rightarrow z^2 + R^2 \approx z^2$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{z^2}$$ ✓

We need or[...]he z axis, as the comp[...]ncel



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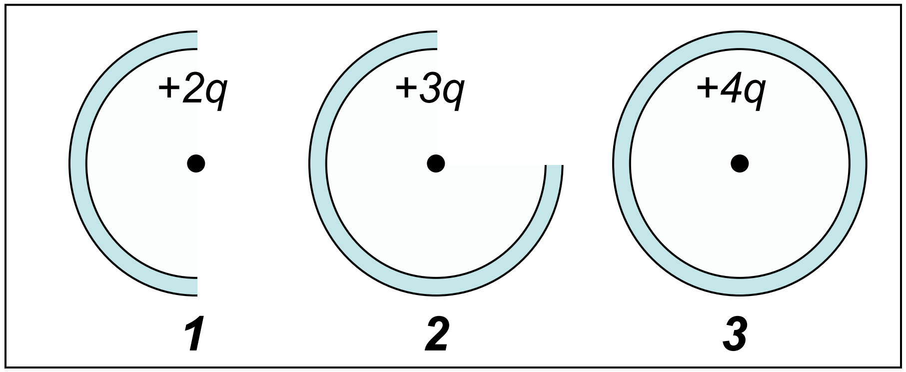
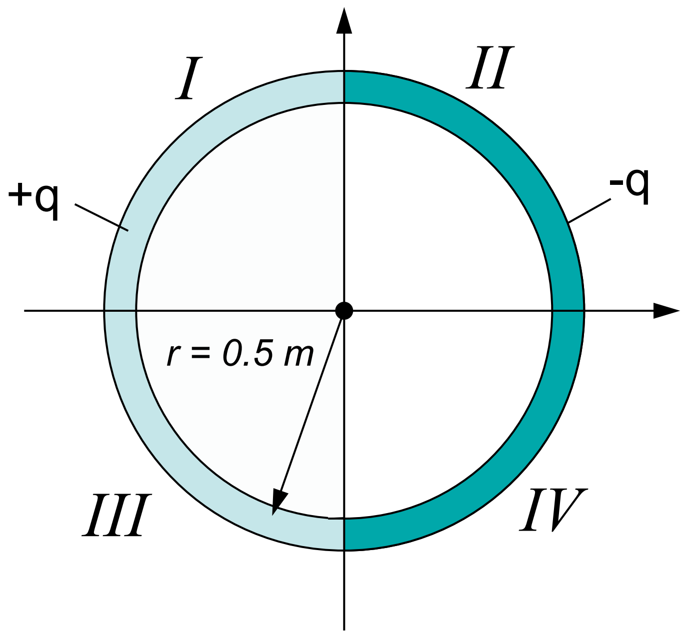
$$E = \frac{Qz}{4\pi\varepsilon_0}\left(z^2 + R^2\right)^{-3/2}$$

A half, 3/4 and full ring are charged as shown below. Rank the scenarios by the magnitude of the electric field at the center.



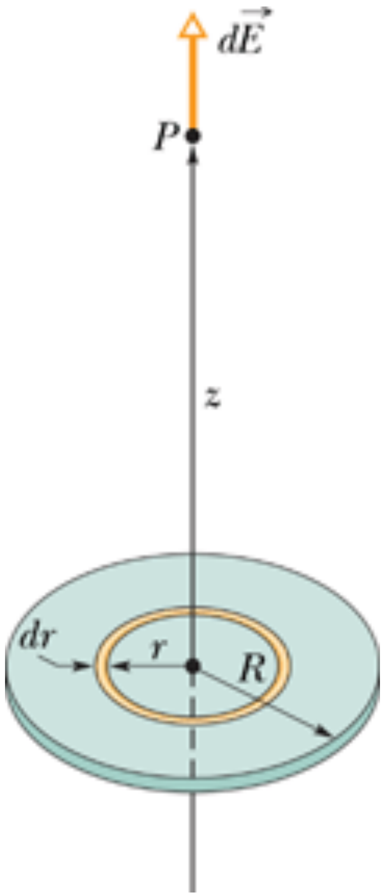
What is the electric field at the center of this hoop?



The Electric CD

Think of a charged disc as a sequence of charged hoops

$$dq = \sigma \cdot dA = \sigma(2\pi r)dr$$



Again consider only points along the z axis so that the electric field points in the z direction

$$E_{hoop}(r) = \frac{Qz}{4\pi\varepsilon_0}\left(z^2 + r^2\right)^{-3/2}$$

$$dE = \frac{z(\sigma 2\pi r dr)}{4\pi\varepsilon_0}\left(z^2 + r^2\right)^{-3/2} dr$$

$$E = \int_0^R \frac{\sigma z}{4\varepsilon_0}\left(z^2 + r^2\right)^{-3/2} 2r dr$$

$$E = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right)$$

In the limit R→∞ then $E \rightarrow \dfrac{\sigma}{2\varepsilon_0}$

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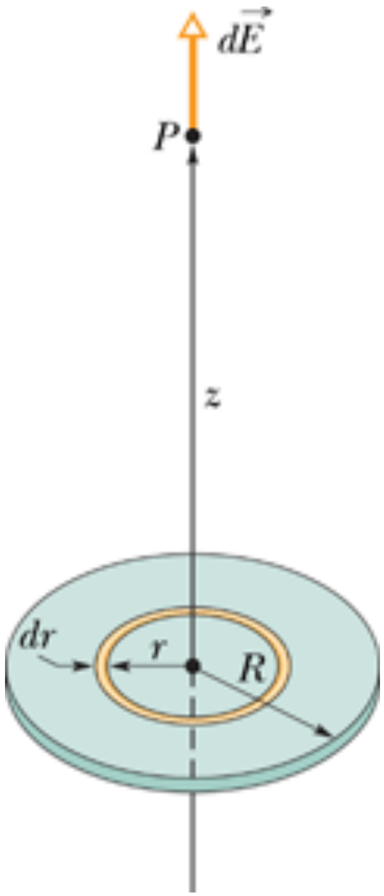


Check your limits

$$z >> R \rightarrow z^2 + R^2 \approx z^2 \quad E = 0 \quad ?!$$

$$\frac{1}{\sqrt{z^2+R^2}} = \frac{1}{z}\left(1 - \frac{1}{2}\frac{R^2}{z^2} + \frac{3}{8}\frac{R^4}{z^4} - \cdots\right)$$

$$E \approx \frac{\sigma}{2\varepsilon_0}\left(1 - \left[1 - \frac{1}{2}\frac{R^2}{z^2}\right]\right)$$

$$E \approx \frac{\sigma}{2\varepsilon_0}\left(\frac{1}{2}\frac{R^2}{z^2}\right) = \frac{1}{4\pi\varepsilon_0}\frac{\sigma\pi R^2}{z^2} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{z^2} \quad ✓$$

$$E = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right)$$

In the limit R→∞ then $E \rightarrow \dfrac{\sigma}{2\varepsilon_0}$

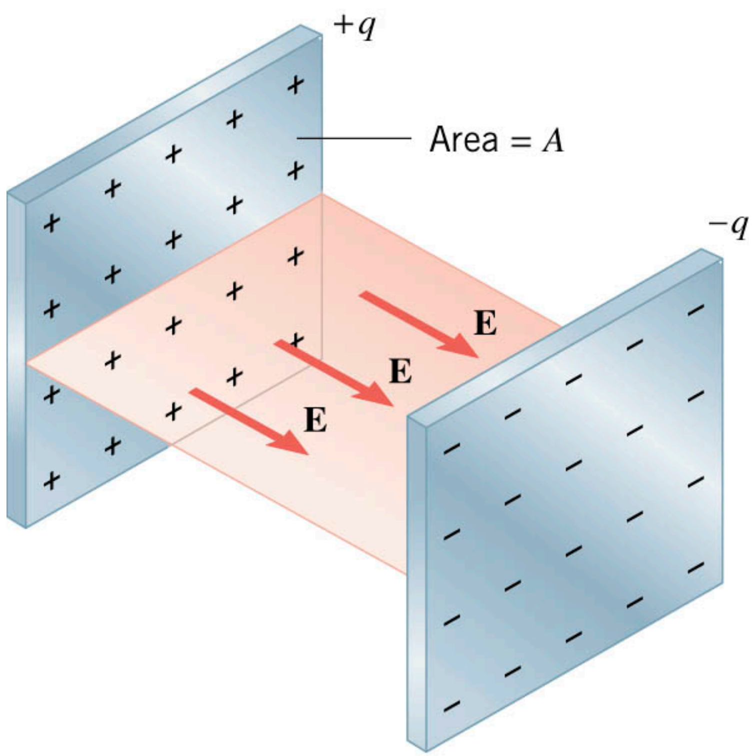
The Parallel-Plate Capacitor

In the limit R→∞ is an important one in real life.

2 conducting parallel plates

One charged with +q and the other charged with -q

Separated by a distance small compared to their size



The resulting electric field between the plates is ***uniform***

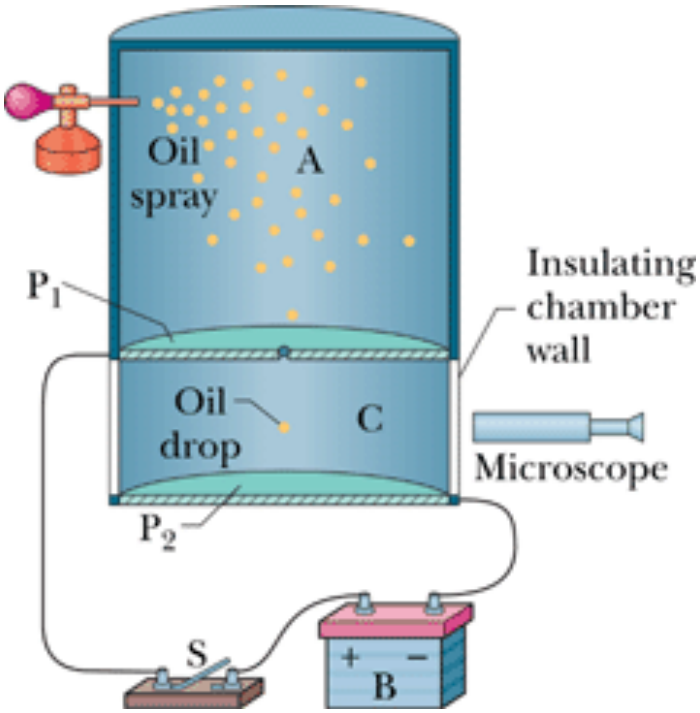
$$E = \frac{\sigma}{\varepsilon_0}$$

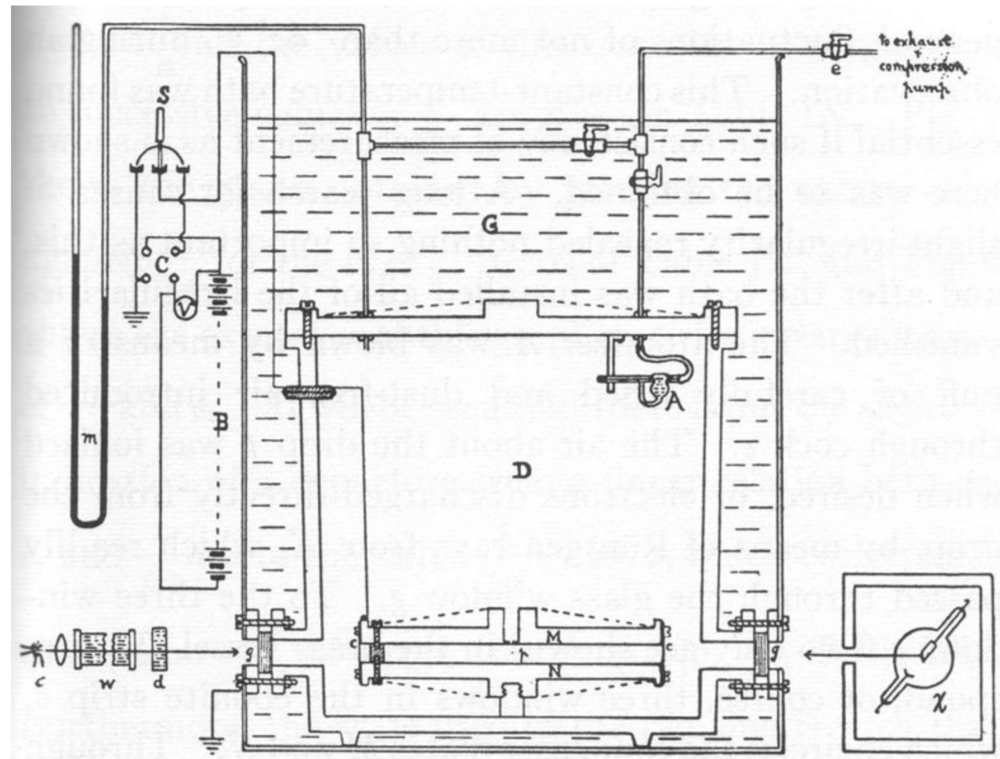
pointing from positive plate to negative plate

The capacitor is widely utilized in electric circuits. . . more on that later

Millikan's Oil Drop Experiment

First accurate determination of the charge on an electron





Very small droplets of oil carry charge of just a few electrons

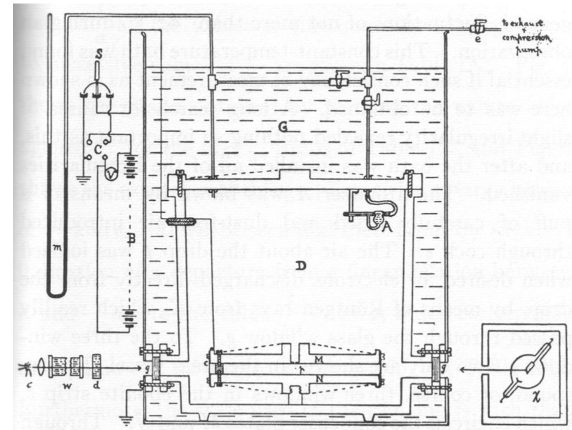
Concept: determine the electric field require to keep a drop "floating"

$qE = mg$ But, how do you know the mass?

Robert Millikan

One of the greatest experimental physicists

1923 Nobel Prize for the oil drop experiment



(1868-1953)
Born: Morristown, IL
U. Chicago 1896-1921
CalTech 1921-1945

Controversy #1 - What drops?

Millikan measured 1.5924(17) x 10^{-19} C
Modern value is 1.6021764 x 10^{-19} C
It took decades to unravel the issue

"Why didn't they discover the new number was higher right away? It's a thing that scientists are ashamed of - this history - because it's apparent that people did things like this: When they got a number that was too high above Millikan's, they thought something must be wrong - and they would look for and find a reason why something might be wrong. When they got a number close to Millikan's value they didn't look so hard. And so they eliminated the numbers that were too far off, and did other things like that." -- Richard Feynman (from *Surely you're joking, Mr. Feynman!*)

Controversy #2 - Harvey Fletcher

Millikan claimed sole authorship for first accurate determination of the electron charge from the oil drop experiment. Millikan's former student, Harvey Fletcher, released a letter in 1981 following his death claiming responsibility for many of the significant developments of the oil drop experiment, but that he made an agreement with Millikan to keep silent about his role in exchange for taking lead authorship on another aspect of the experiment.

A disk (outer radius *2R*) with a hole in the middle (inner radius *R*) has a net positive charge *+q*. A rod with net charge *–q* and length *2R* lies along the z axis, which is the symmetry axis of the disk as shown. Find the net electric field at Point *P* on the *z* axis. What is the form of the field if z>>R?