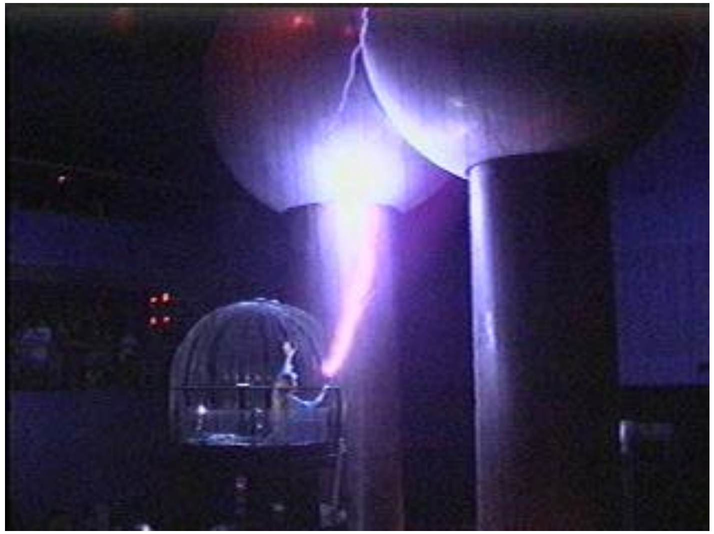
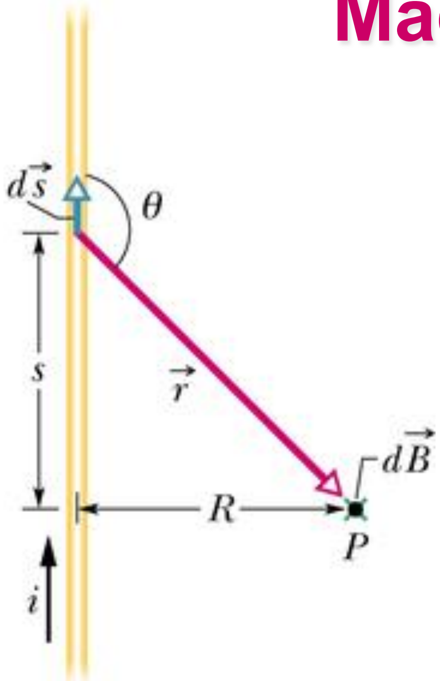
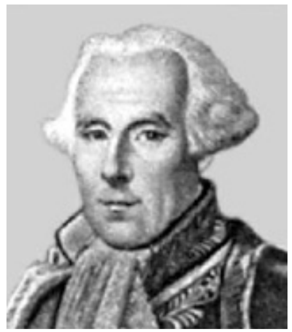


Lecture 27

Magnetic Fields Due to Currents III



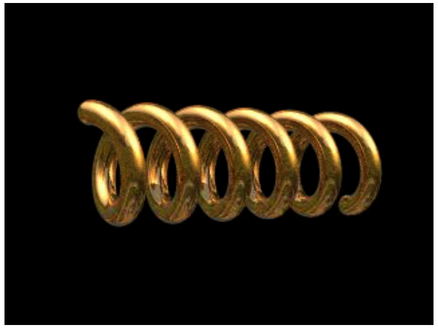
Jean-Baptiste Biot (1774-1862)



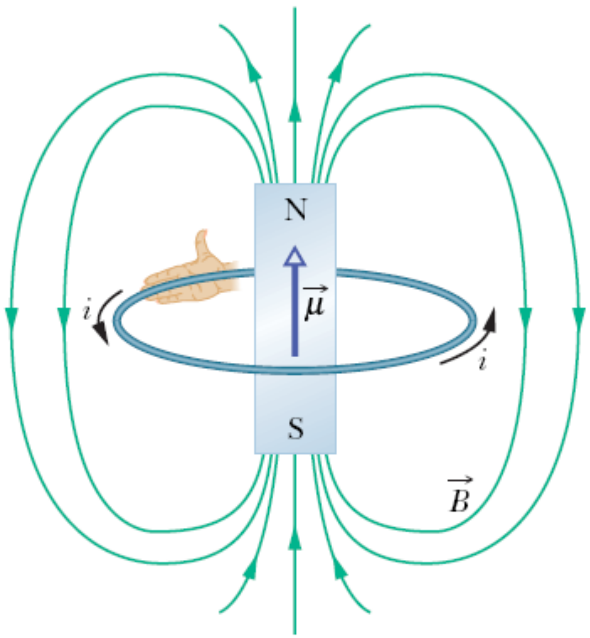
Felix Savart (1791–1841)

Agenda

- Magnetic field inside a Solenoid and a Toroid



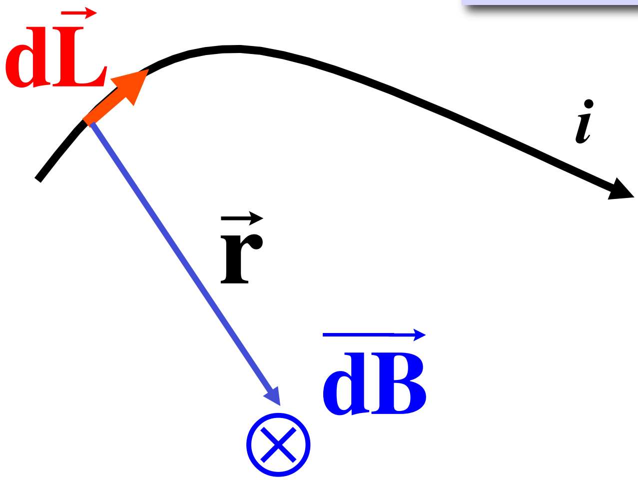
- A current-carrying coil as a magnetic dipole



Recap

Biot-Savart Law for B-Fields

$$\mathbf{d}\vec{\mathbf{B}} = \frac{\mu_0}{4\pi} \frac{i\mathbf{d}\vec{\mathbf{L}} \times \hat{\mathbf{r}}}{r^2}$$

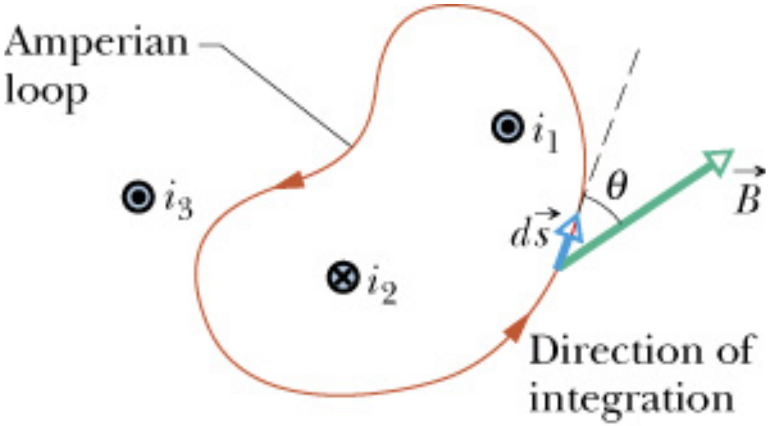


Uses the
Right hand Thumb Rule!

Recap

Ampere's Law for B-Fields

Analogue of Gauss' law of electrostatics for magnetic fields!



If you have a lot of symmetry, knowing the circulation of B allows you to know B.

$$\oint_{\text{LOOP}} \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{enclosed}}$$

$$\oint_{\text{loop}} \vec{B} \cdot d\vec{s} = \mu_0 (i_1 - i_2)$$

9 Figure 29-31 shows four circular Amperian loops (a, b, c, d) and, in cross section, four long circular conductors (the shaded regions), all of which are concentric. Three of the conductors are hollow cylinders; the central conductor is a solid cylinder. The currents in the conductors are, from smallest radius to largest radius, 4 A out of the page, 9 A into the page, 5 A out of the page, and 3 A into the page. Rank the Amperian loops according to the magnitude of $\oint \vec{B}\cdot d\vec{s}$ around each, greatest first.

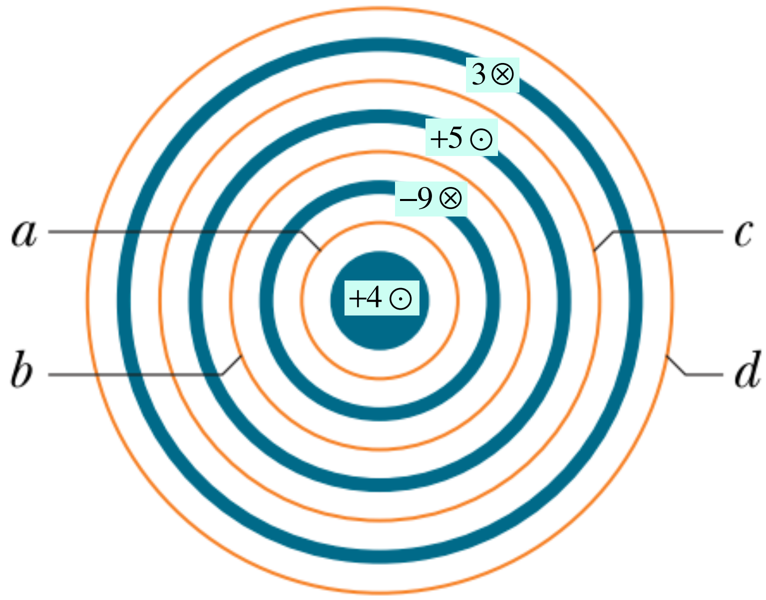


Fig. 29-31 Question 9.

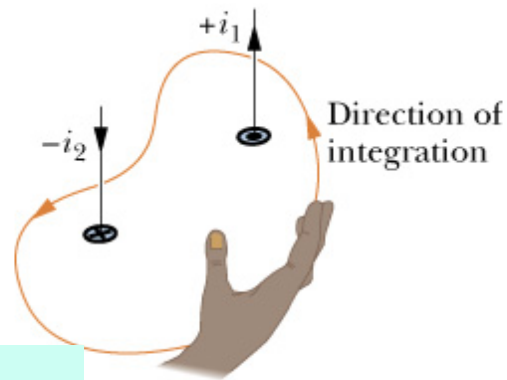
$$\oint_a \propto |4| = 4$$

$$\oint_b \propto |4-9| = 5$$

$$\oint_c \propto |4-9+5| = 0$$

$$\oint_d \propto |4-9+5-3| = 3$$

$$\oint_{\text{LOOP}} \vec{B}\cdot d\vec{s} = \mu_0 i_{\text{enclosed}}$$



$$\oint_b > \oint_a > \oint_d > \oint_c = 0$$

9 Figure 29-31 shows four circular Amperian loops (a, b, c, d) and, in cross section, four long circular conductors (the shaded regions), all of which are concentric. Three of the conductors are hollow cylinders; the central conductor is a solid cylinder. The currents in the conductors are, from smallest radius to largest radius, 4 A out of the page, 9 A into the page, 5 A out of the page, and 3 A into the page. Rank the Amperian loops according to the magnitude of $\boldsymbol{B}$ around each, greatest first.

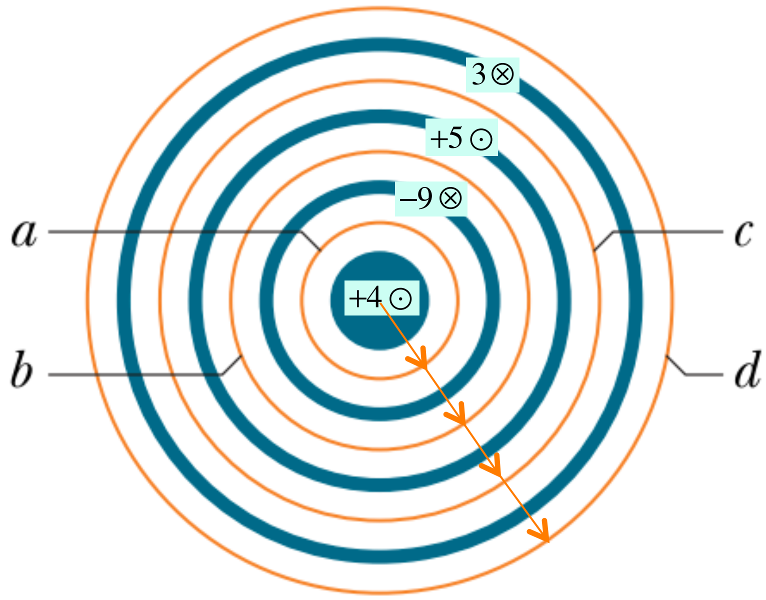


Fig. 29-31 Question 9.

$$B_a \propto |4|/1 = 4 = 16/4$$

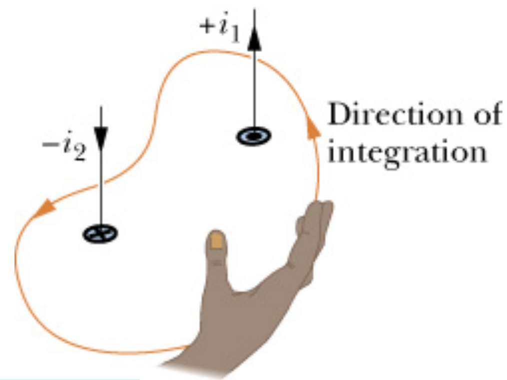
$$B_b \propto |4-9|/2 = 5/2 = 10/4$$

$$B_c \propto |4-9+5|/3 = 0$$

$$B_d \propto |4-9+5-3|/4 = 3/4$$

$$B = \frac{\mu_0 i_{\text{enclosed}}}{2\pi r}$$

1/r Law too



$$B_a > B_b > B_d > B_c = 0$$

29.5: Solenoids and Toroids:

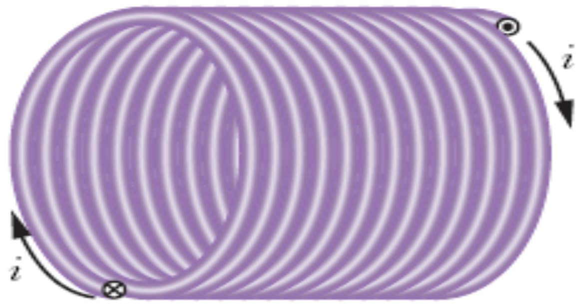


Fig. 29-16 A solenoid carrying current i.

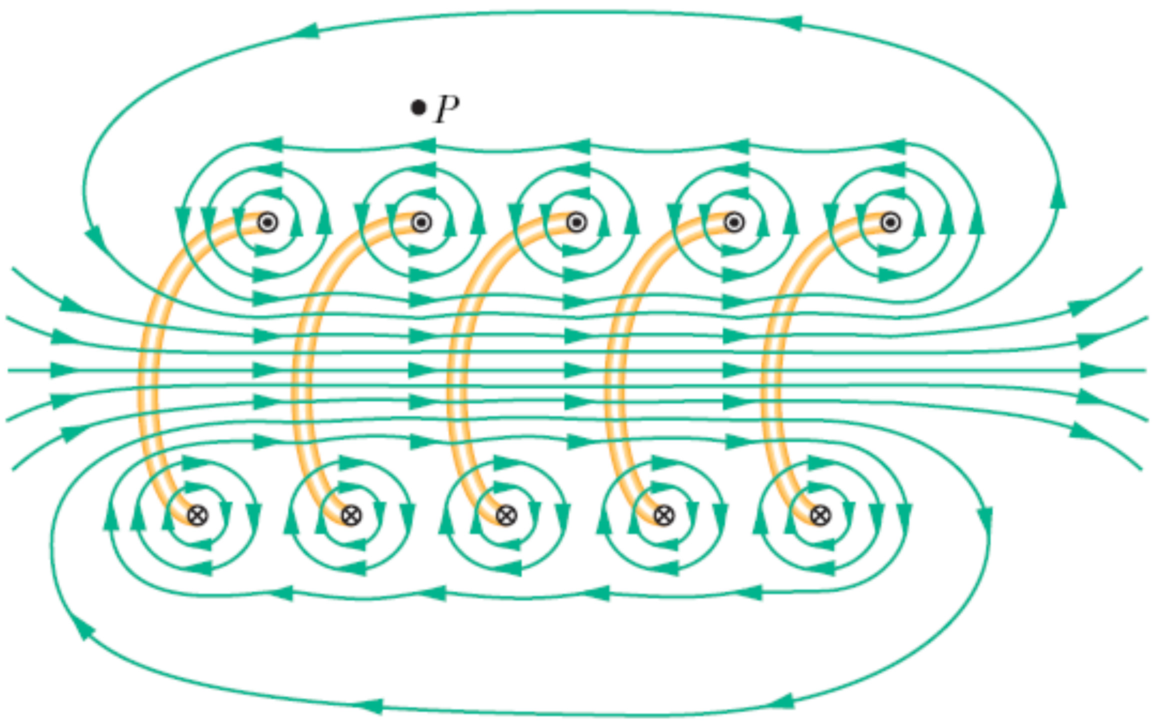
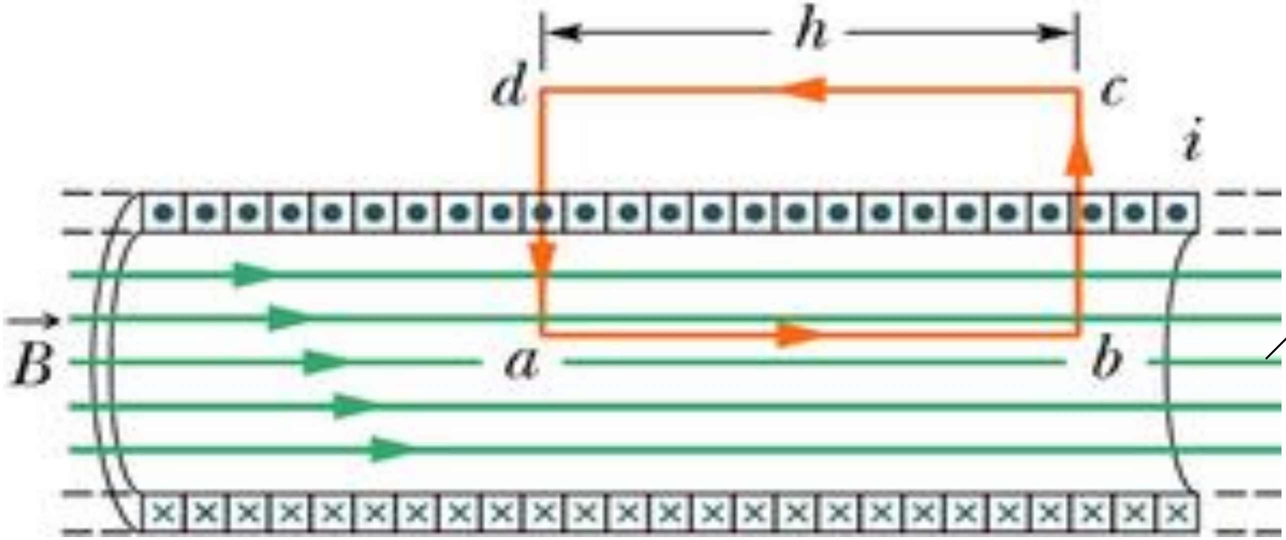


Fig. 29-17

Solenoids: Compute the B-Field Inside

Ideal (Infinitely long and tightly woven) Solenoid



Uniform B field!

$$\oint \vec{B}\cdot d\vec{s} = \int_a^b \vec{B}\cdot d\vec{s} + \int_b^c \vec{B}\cdot d\vec{s} + \int_c^d \vec{B}\cdot d\vec{s} + \int_d^a \vec{B}\cdot d\vec{s}.$$

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 i_{\text{enc}} \quad \text{(Ampere's Law)}$$

$$\oint \vec{B}\cdot d\vec{s} = Bh + 0 + 0 + 0$$

$$i_{\text{enc}} = iN_h = i(N/L)h = inh$$

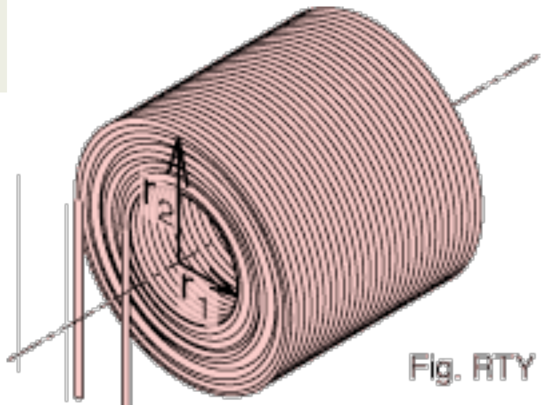
$$\oint \vec{B}\cdot d\vec{s} = \mu_0 i_{\text{enc}} \Rightarrow Bh = \mu_0 inh \Rightarrow B = \mu_0 in$$

n = N/L is turns per unit length.

Sample Problem

The field inside a solenoid (a long coil of current)

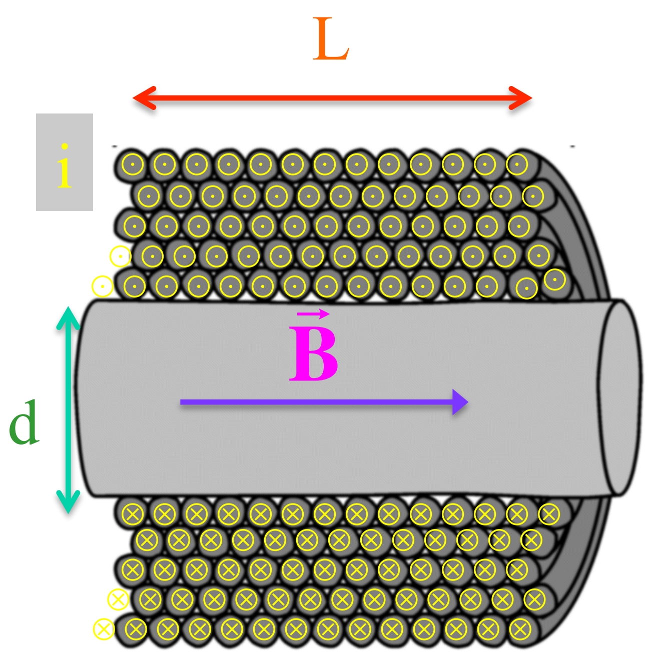
A solenoid has length $L = 1.23$ m and inner diameter $d = 3.55$ cm, and it carries a current $i = 5.57$ A. It consists of five close-packed layers, each with 850 turns along length L. What is B at its center?



A radially thick, multilayer solenoid.

KEY IDEA

The magnitude B of the magnetic field along the solenoid's central axis is related to the solenoid's current i and number of turns per unit length n by Eq. 29-23 ($B = \mu_0 in$).



Calculation: Because B does not depend on the diameter of the windings, the value of n for five identical layers is simply five times the value for each layer. Equation 29-23 then tells us

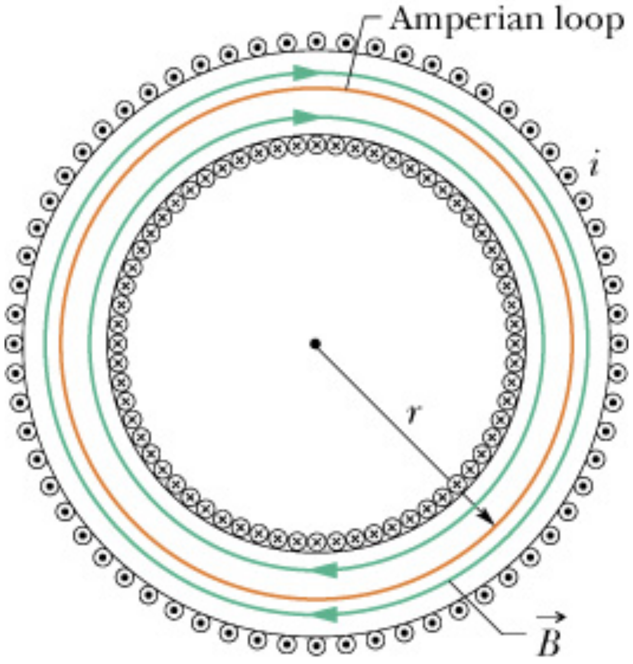
$$B = \mu_0 in = (4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(5.57\ \text{A})\frac{5 \times 850\ \text{turns}}{1.23\ \text{m}}$$

$$= 2.42 \times 10^{-2}\ \text{T} = 24.2\ \text{mT}. \qquad \text{(Answer)}$$

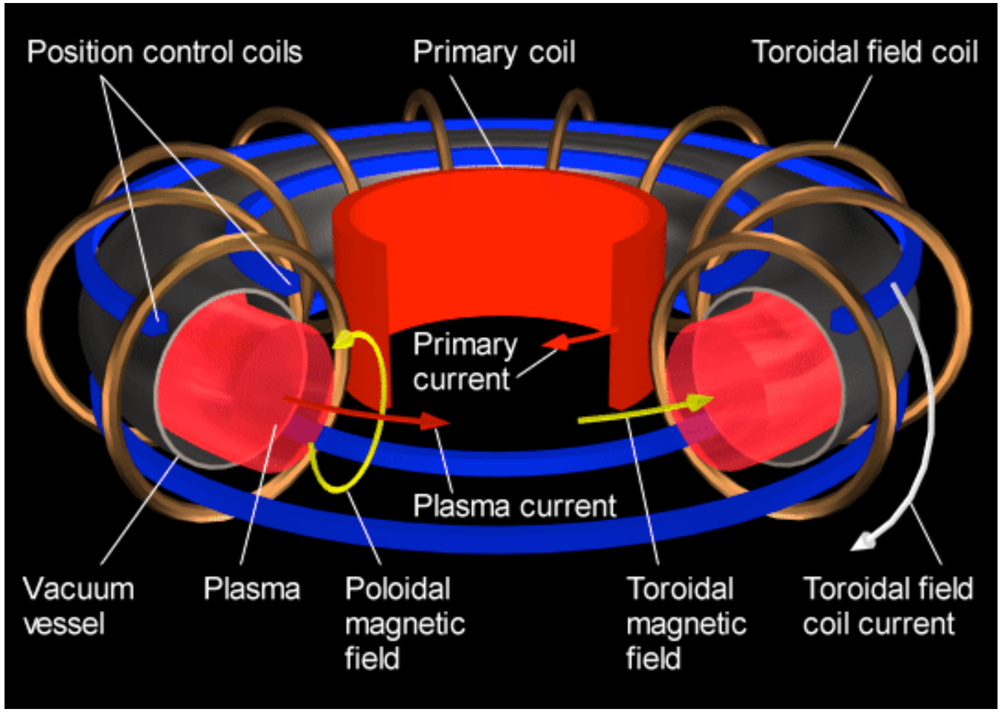
To a good approximation, this is the field magnitude throughout most of the solenoid.

Magnetic Field in a Toroid “Doughnut” Solenoid

(a)

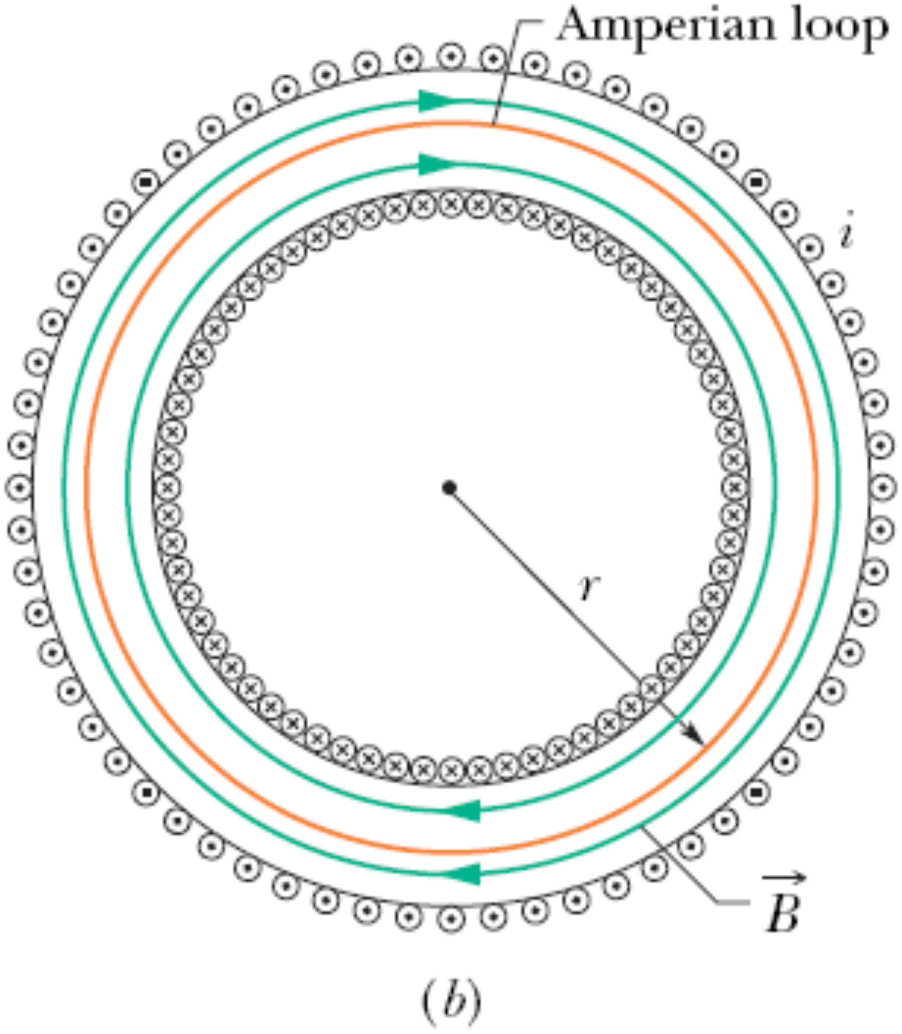


(b)



Toriod Fusion Reactor:
Power NYC For a Day on a Glass of H_2O

29.5: Magnetic Field of a Toroid:



$$\oint \vec{B}\cdot d\vec{s} = \mu_0 i_{enc} \quad \text{(Ampere's Law)}$$

$$\oint \vec{B}\cdot d\vec{s} = B2\pi r$$

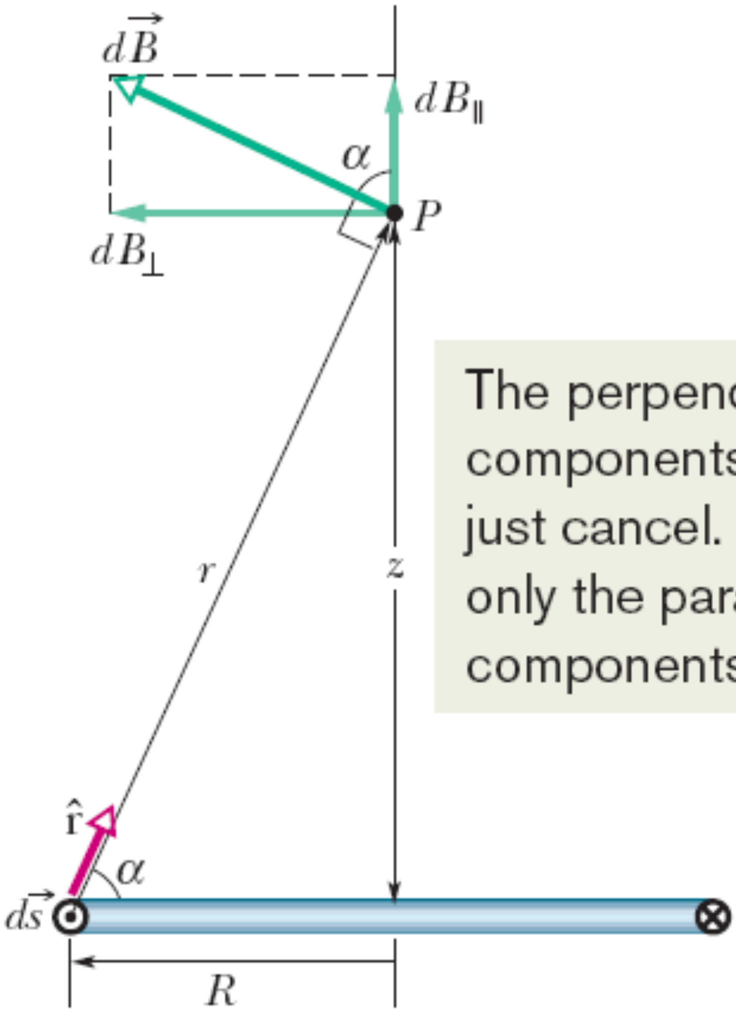
$$i_{enc} = Ni$$

$$(B)(2\pi r) = \mu_0 iN,$$

i = the current in the toroid windings
N = the total number of turns.

$$B = \frac{\mu_0 iN}{2\pi}\frac{1}{r} \quad \text{(toroid)}.$$

29.6: A Current Carrying Coil as a Magnetic Dipole:



The perpendicular components just cancel. We add only the parallel components.

$$dB = \frac{\mu_0}{4\pi}\frac{i\,ds\sin 90^\circ}{r^2}.$$

$$dB_\parallel = dB\cos\alpha. \quad = \frac{\mu_0 i\cos\alpha\, ds}{4\pi r^2}.$$

$$r = \sqrt{R^2 + z^2}$$

$$\cos\alpha = \frac{R}{r} = \frac{R}{\sqrt{R^2 + z^2}}.$$

$$dB_\parallel = \frac{\mu_0 i R}{4\pi(R^2 + z^2)^{3/2}}\,ds.$$

$$B = \int dB_\parallel$$

$$= \frac{\mu_0 i R}{4\pi(R^2 + z^2)^{3/2}}\int ds$$

$$B(z) = \frac{\mu_0 i R^2}{2(R^2 + z^2)^{3/2}}.$$

For small $z << R$

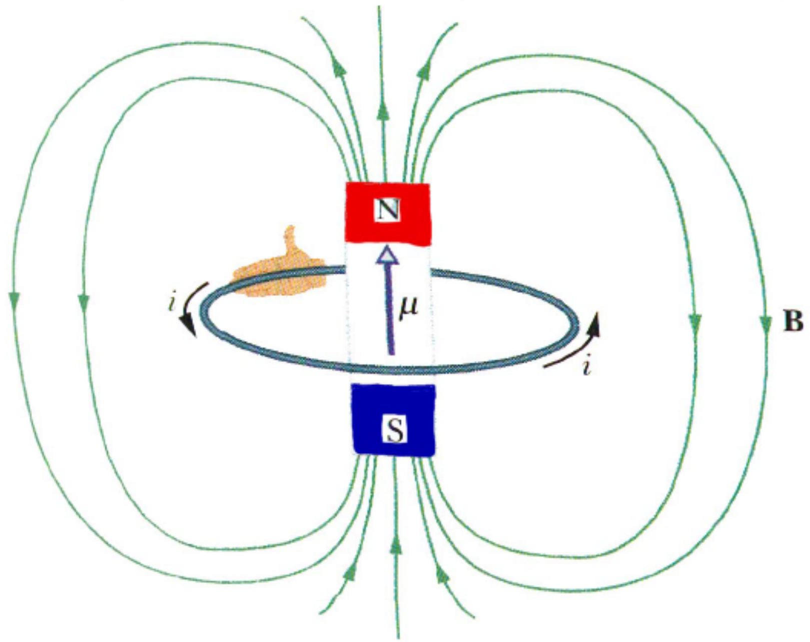
$$B(z) = \frac{\mu_0 i R}{2R^3\left(1 + z^2 / R^2\right)^{3/2}} \cong \frac{\mu_0 i}{2R^2}$$

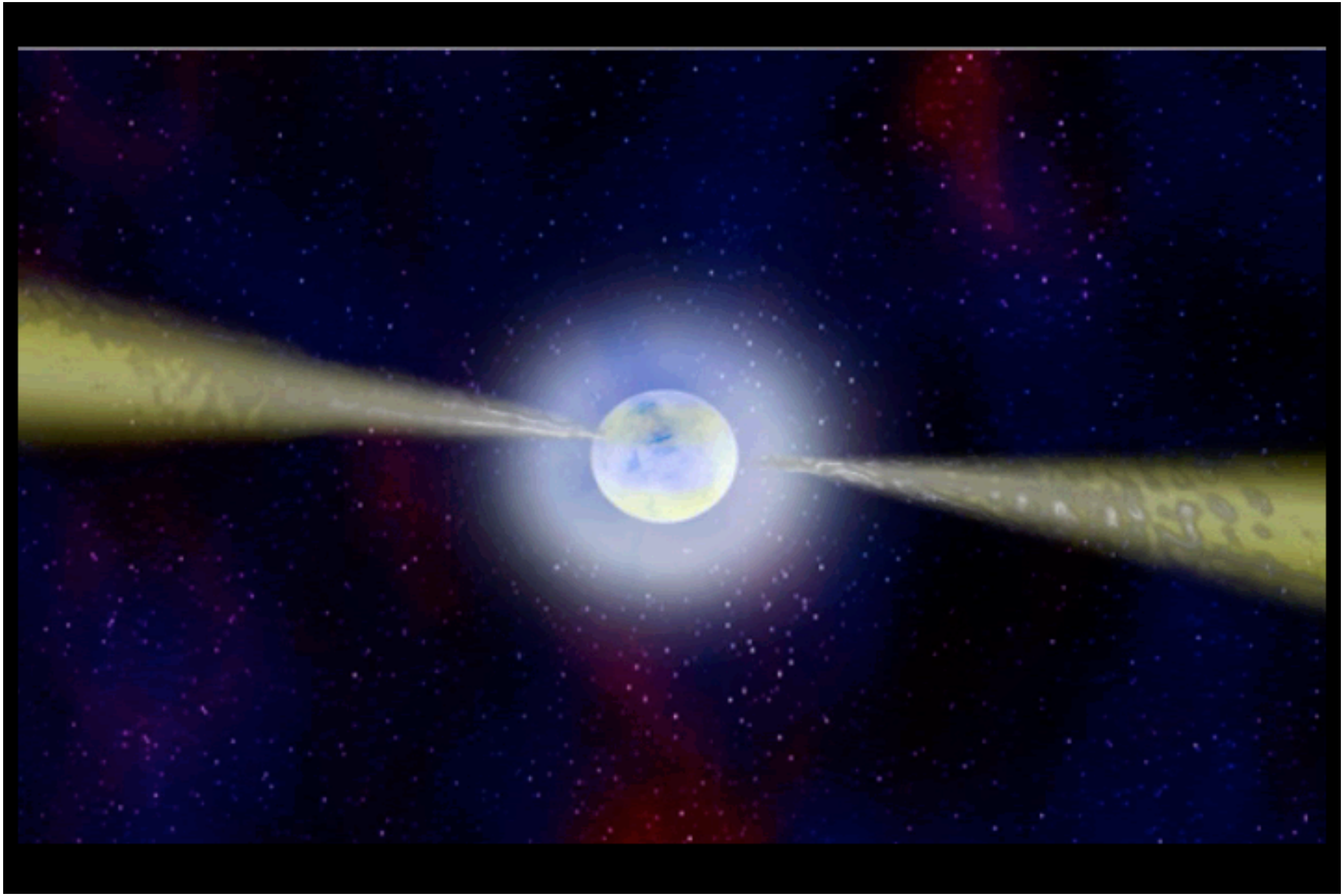
$1 / R^2$ Law! Double the R one fourth the field.

μ dipole moment

On the other hand,

If, $z >> R$, $$B = \frac{\mu_0}{2}\frac{i R^2}{z^3} = \frac{\mu_0}{2\pi}\frac{i \text{ Area}}{z^3} = \frac{\mu_0}{2\pi}\frac{\mu}{z^3}$$

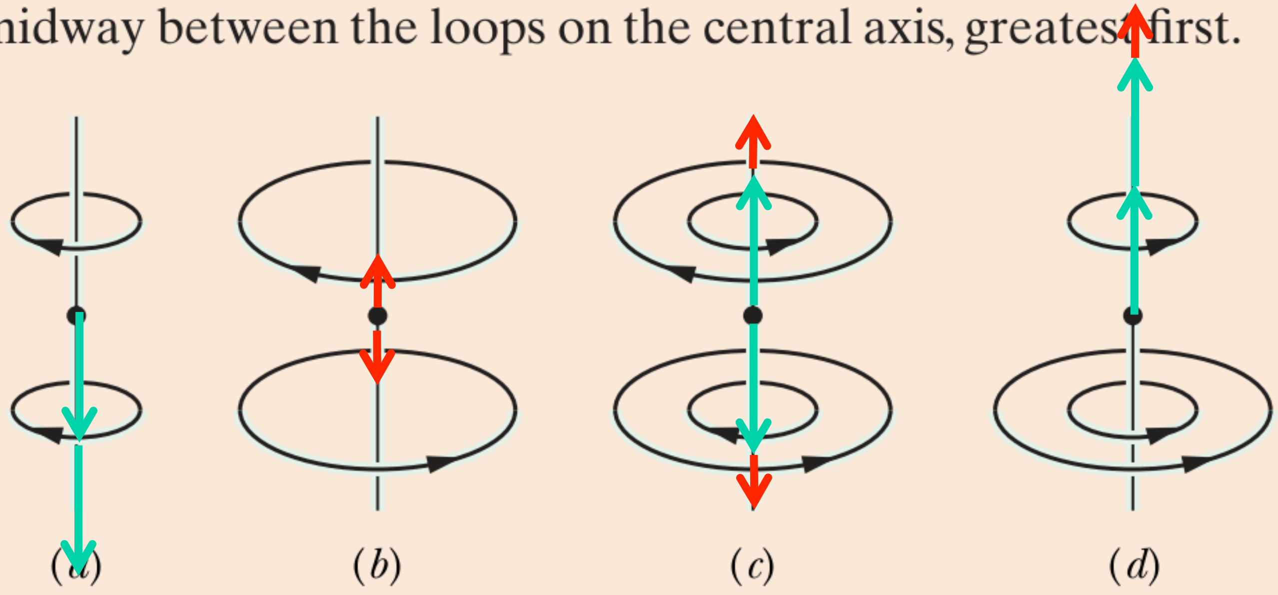


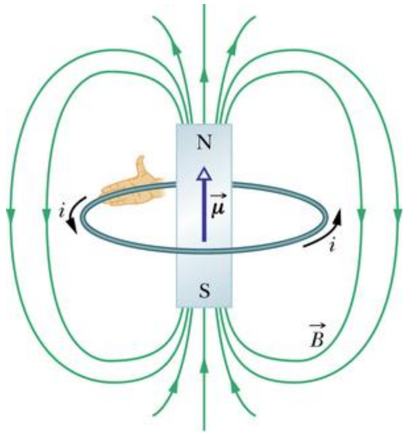


Neutron Star a Large Magnetic Dipole $B_{surface} = 10^{10}$ Tesla

CHECKPOINT 3

The figure here shows four arrangements of circular loops of radius *r* or 2*r*, centered on vertical axes (perpendicular to the loops) and carrying identical currents in the directions indicated. Rank the arrangements according to the magnitude of the net magnetic field at the dot, midway between the loops on the central axis, greatest first.





$$B_d > B_a > B_b = B_c = 0$$

For small $z << R$

$$B(z) \cong \frac{\mu_0 i}{2R^2}$$

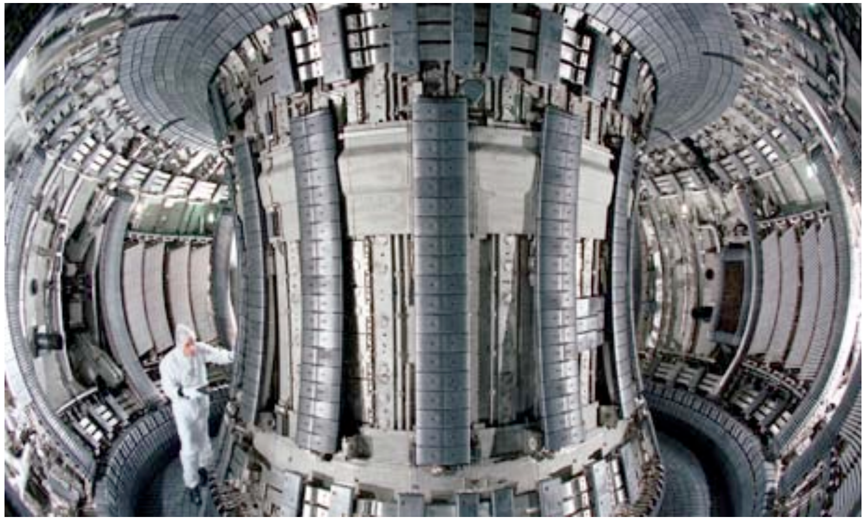
$1/R^2$ Law!

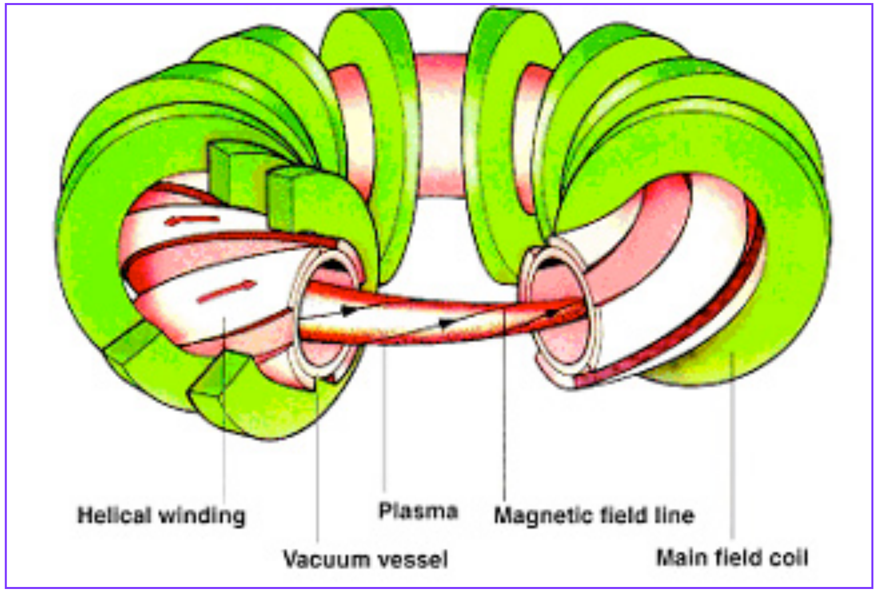
Double the R $\Rightarrow$ one 4th the field.

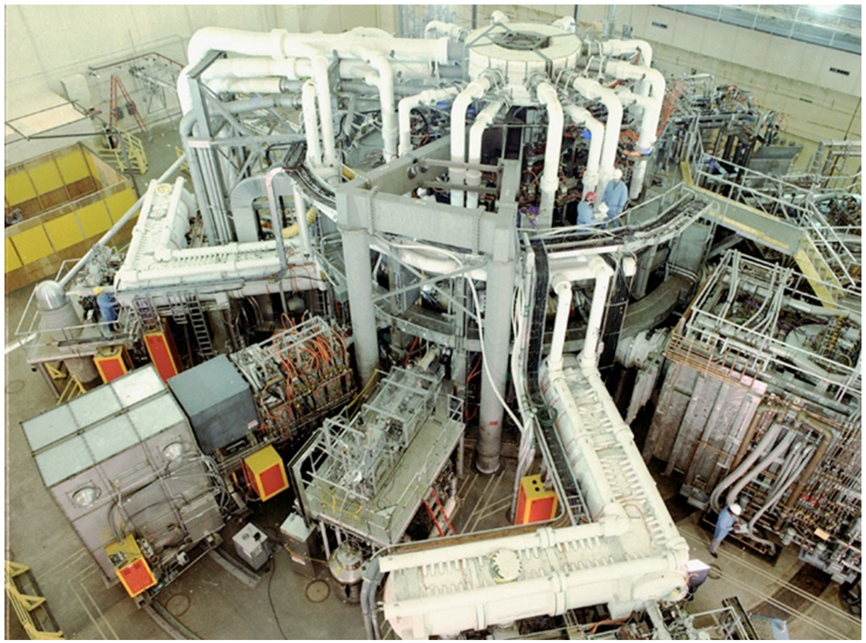
Summary

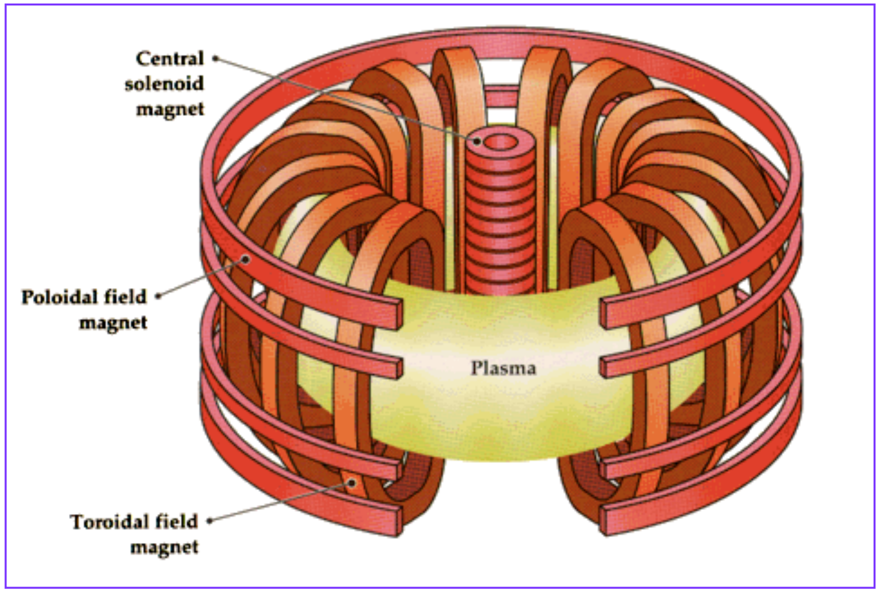
- We used Ampere's law to determine the magnetic field inside a solenoid and a toroid.
- We used Biot-Savart's law to show that a current-carrying coil behaves like a magnetic dipole.

Solenoid Fusion Reactors









Lockheed Martin's new fusion reactor might change humanity forever

Jesus Diaz
Filed to: AWESOME 10/15/14 9:41am

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