Test 3 - Physics 2113 - Fall 2016

November 9, 2016

Last Name: Key_______________ First name:_________________

Sec. 1 MWF 8:30am (Rupnik) Sec. 2 MWF 10:30am (Stadler) Sec. 3 MWF 12:30pm (Blackmon)

Sec. 4 MWF 1:30pm (Launey) Sec. 5 MWF 2:30pm (Zuniga-Hansen)

Sec. 6 TuTh 12:00pm (Wilde) Sec. 7 TuTh 1:30pm (Zuniga-Hansen)

Be sure to write your name and circle your section.

Answer all 5 questions (6 points each) and 3 problems (25 points each).

Please read the questions carefully.

You may use scientific or graphing calculators.

You may detach and use the formula sheet provided at the back of this test. No other reference materials are allowed.

You are strictly forbidden from having access to any electronic communications device during a test. This includes cell phones, pagers, smartphones and tablet or notebook computers. You may not use calculator software on such a device during the test. Any student found with such a device will be assumed to be using it to cheat, and will be reported to the Dean of Students for disciplinary action. Any student who observes another student using such a device during the test should notify the instructor or proctor immediately.

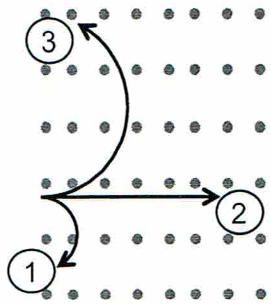
Please use clear, complete sentences if explanations are asked for.

Some questions are multiple choice. You should work these problems starting with the basic equation listed on the formula sheet and write down the steps. Although the work will not be graded, this will help you make the correct choice and be able to determine if your thinking is correct. ***Be sure to mark your final answer clearly.***

On problems that are not multiple choice, you ***must show all of your work***. No credit will be given for an answer without explanation or work. These will be graded in full, and you are expected to show all relevant steps that lead to your answer.

YOU GET 60 min (1 hr)

Question 1: [6 pts] The figure shows 3 particles with the same mass and the same velocity entering a uniform magnetic field (with direction out of the page).



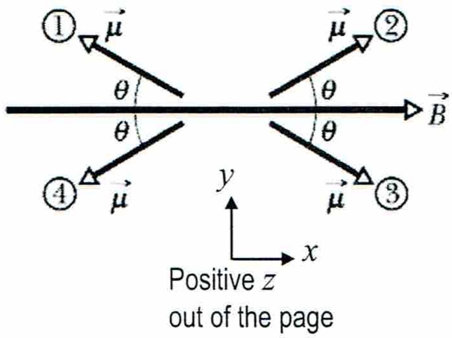
(a) Rank the charges q (not magnitude $|q|$), greatest first.

- ● $q_1 > q_2 > q_3$
- ○ $q_2 > q_3 > q_1$
- ○ $q_1 > q_3 > q_2$
- ○ $q_3 > q_1 > q_2$
- ○ $q_2 > q_1 > q_3$
- ○ $q_3 > q_2 > q_1$

(b) If the speed of particle 3 increases by a factor of two, its period

- ○ Increases
- ● Stays the same
- ○ Decreases

Question 2: [6 pts] The figure shows four orientations of magnetic dipole moment with moment μ in a uniform magnetic field B. The dipole moment μ rotates from orientation 1 to orientation 2.



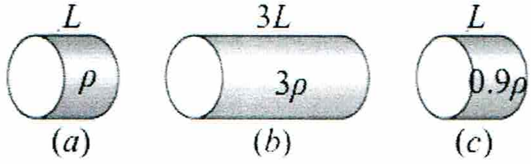
(a) The magnetic potential energy

- ● Decreases
- ○ Stays the same
- ○ Increases

(b) The direction of the torque for orientation 3 is

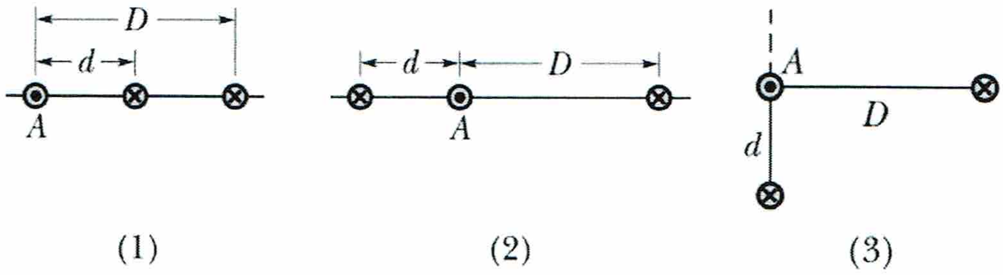
- ○ Positive x
- ○ Negative x
- ○ Positive y
- ○ Negative y
- ● Positive z
- ○ Negative z
- ○ Zero

Question 3: [6 pts] Three cylindrical wires, of the same diameter, are depicted below. Their resistivities and lengths are ρ and L (wire a), 3ρ and $3L$ (wire b), and 0.9ρ and L (wire c). Suppose each wire has the same potential difference placed across its terminals. Rank the wires according to the power dissipated in them, greatest first.



- ○ $b > a > c$
- ● $c > a > b$
- ○ $a > b > c$
- ○ $c > b > a$
- ○ $a = b > c$

Question 4: [6 pts] The figure below shows three arrangements of three long straight wires carrying equal currents directly into or out of the page, where the distances $d = 1\text{m}$ and $D = 2\text{m}$.



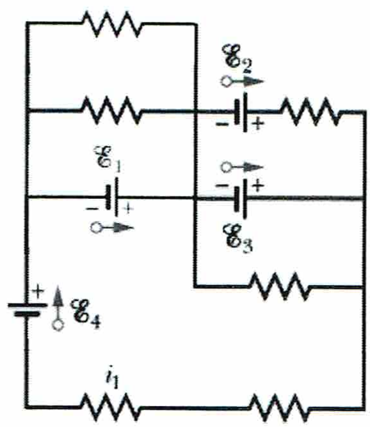
(a) Rank the arrangements according to the *magnitude* of the net force on wire A due to the currents in the other wires, greatest first.

- ○ 3 > 1 > 2
- ○ 3 > 2 > 1
- ○ 2 > 3 > 1
- ○ 1 > 2 > 3
- ● 1 > 3 > 2

(b) In arrangement 3, is the angle between the net force on wire A and the dashed line equal to, less than, or more than 45°?

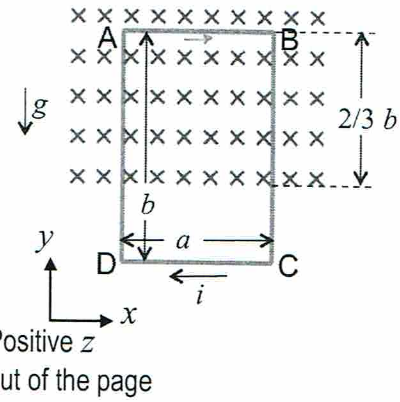
- ○ = 45°
- ● < 45°
- ○ > 45°

Question 5: [6 pts] Refer to the circuit in the figure, which contains resistors and ideal batteries with $\mathscr{E} = 4$ V each. Each resistor has resistance 3Ω. What is the power being delivered by battery 4?



- ○ 4 W
- ● 8 W
- ○ 12 W
- ○ 24 W
- ○ 48 W
- ○ 54 W

Problem 1: [25 pts] A rectangular conducting loop with sides a=0.1m and b = 0.3 m, and with clockwise current i = 5 A *hangs freely* (in equilibrium) in the presence of a uniform horizontal magnetic field B = 0.04 T, which has direction into the page, as illustrated in the figure. The gravitational force is depicted going in the $-y$ direction, along with Earth's gravitational acceleration g.



(a) (12 points) In unit-vector notation, find the following magnetic forces (convention is positive x to the right, positive y up, and positive z out of the page).

1) Magnetic force on segment AB:

$\vec{F} = i\vec{L}\times\vec{B}$

$|\vec{F}| = iaB = (5A)(0.1m)(0.04T) = 0.02\ N \Rightarrow \vec{F}_{AB} = (0.02N)\hat{j}$

2) Magnetic force on segment CD:

$F_{CD} = 0$

3) Magnetic force on segment AD:

$F_{AD} = i\left(\frac{2}{3}b\right)B = (5A)(0.2m)(0.04T) = 0.04\ N \Rightarrow \vec{F}_{AD} = -(0.04N)\hat{i}$

4) Magnetic force on segment BC:

$F_{CD} = i\left(\frac{2}{3}b\right)B = 0.04\ N \Rightarrow \vec{F}_{BC} = (0.04N)\hat{i}$

5) Net magnetic force on the loop:

$\vec{F}_{net} = \vec{F}_{AB} + \vec{F}_{CD} + \vec{F}_{AD} + \vec{F}_{CD} = (0.02N)\hat{j}$

[or] $= (0.02N)\hat{j} + 0 + (-0.04N)\hat{i} + (0.04N)\hat{i} = (0.02N)\hat{j}$

(b) (8 points) Calculate the mass of the loop.

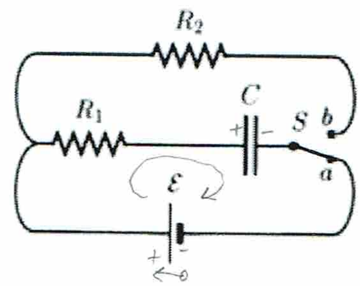
$\vec{F}_{net} + m\vec{g} = 0 \Rightarrow (0.02N)\hat{j} - mg\hat{j} = 0 \Rightarrow m = \frac{(0.02N)}{g} = 0.002\ kg$

(c) (5 points) What would be the direction of the current in wire CD if the direction of the magnetic field were reversed (out of the page) and the loop is again in equilibrium?

Positive x

Problem 2: [25 pts] Consider the figure of the *RC* circuit depicted below. Each resistor has resistance 3 Ω. The ideal battery has emf $\mathcal{E} = 12.0$ V. Express your answers in terms of the capacitance *C*.

At time $t = 0$, the switch *Sa* is closed:



(a) (5 points) What is the time constant τ_a?

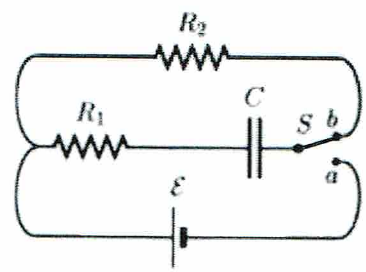
$\tau_a = R_1 C = (3\Omega) C$

(b) (5 points) After a long time, what is the potential difference across *C*?

after a long time, current in circuit = 0

$\Rightarrow \mathcal{E} - V_C = 0 \Rightarrow V_C = \mathcal{E} = 12V$

The switch is then moved to *Sb*. Immediately after this happens,



(c) (5 points) What is the current through R_2?

Find equivalent resistance $R_s = R_1 + R_2 = 6\Omega$

at $t=0$: $V_C(t=0) = \mathcal{E}$

$$i = \frac{V_C(t=0)}{R_S} = \frac{\mathcal{E}}{R_S} = \frac{12V}{6\Omega} = 2A$$

(d) (5 points) What is the time constant τ_b?

$\tau_b = R_s C = (6\Omega) C$

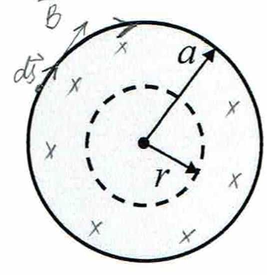
(e) (5 points) What is the charge on *C* after time $t = 2\tau_b$?

$q(t) = q_0 e^{-t/\tau_b}$, with $t = 2\tau_b$ and $q_0 = C V_C(t=0) = C\mathcal{E}$

$q(t) = C\mathcal{E} e^{-2} = C(12V)\, 0.14 = (1.62V)C$

Problem 3: [25 pts] The current density $\vec{J}$ inside a long, solid, cylindrical wire of radius a is along the central axis with direction into the page, and its magnitude varies with radial distance r from the axis according to $J = J_0\, r/a$. The figure below depicts a cross-section of the cylindrical wire.

Express your answers in terms of μ_0, J_0, and a.



(a) (8 points) What is the total current flowing through the cross-section of the cylindrical wire?

$$i = \int \vec{J}\cdot d\vec{A} = \int J\,dA = \int_0^a \left(J_0 \frac{r}{a}\right) 2\pi r\,dr$$

($\vec{J}$ and $d\vec{A}$ in the page)

$$= \frac{J_0}{a} 2\pi \int_0^a r^2 dr = \frac{J_0}{a} 2\pi \left.\frac{r^3}{3}\right|_0^a = \frac{J_0}{a} 2\pi \frac{a^3}{3}$$

$$i = \frac{2\pi}{3} J_0 a^2$$

(b) (8 points) What is the magnitude of the magnetic field at $r = a$?

Use Ampère's Law: $\oint \vec{B}\cdot d\vec{s} = \mu_0 i_{enc}$

Amperian loop: circle with radius $r = a$, direction of integration: clockwise

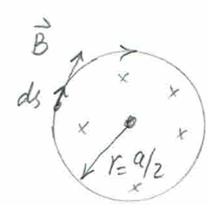
(1) $\oint \vec{B}\cdot d\vec{s} = \oint B\,ds = B \oint ds = B(2\pi a)$

($\vec{B}$ and $d\vec{s}$ in same direction) (B: same magnitude along loop)

(2) $\mu_0 i_{enc} = \mu_0 \frac{2\pi}{3} J_0 a^2$ (from part (a) above)

$\Rightarrow B(2\pi a) = \mu_0 \frac{2\pi}{3} J_0 a^2 \Rightarrow B = \mu_0 J_0 \frac{a}{3}$

(c) (9 points) What is the magnitude of the magnetic field at $r = a/2$?



$\oint \vec{B}\cdot d\vec{s} = \mu_0 i_{enc}$, Amperian loop with $r = \frac{a}{2}$

(1) $\oint \vec{B}\cdot d\vec{s} = B \oint ds = B\left(2\pi \frac{a}{2}\right) = B\pi a$

(2) $\mu_0 i_{enc} = \mu_0 \int \vec{J}\cdot d\vec{A} = \mu_0 \int_0^{a/2} \left(J_0 \frac{r}{a}\right) 2\pi r\,dr = \mu_0 \frac{J_0}{a} 2\pi \left.\frac{r^3}{3}\right|_0^{a/2} = \mu_0 \frac{\pi}{12} J_0 a^2$

(only current enclosed in loop)

$\Rightarrow B\pi a = \mu_0 \frac{\pi}{12} J_0 a^2$

$\Rightarrow B = \mu_0 J_0 \frac{a}{12}$