

# Lecture 35

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19 Nov 2014

review of Maxwell's equations

Gauss' law for <sup>electricity</sup> ~~magnetism~~

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$$

Gauss law for magnetism

$$\oint \vec{B} \cdot d\vec{A} = 0$$

Faraday law of induction:

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

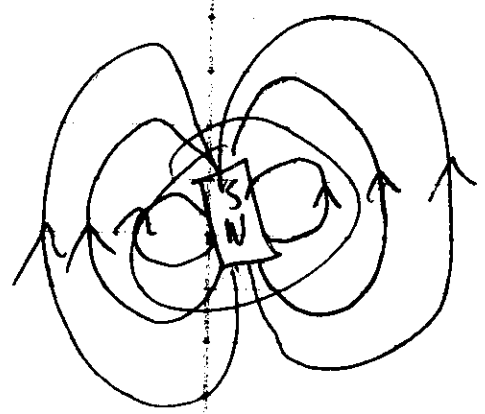
changing  
B-field  
induces magnetic  
field

Ampere-Maxwell law:

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{enc}$$

changing E-field  
& current induces  
magnetic field

Magnetic field of earth -  
can approximate as a bar magnet



geomagnetic poles are  
misaligned w/ geographical  
poles.

Field is due to metal core  
of Earth. <sup>Hot</sup> Inner core is  
solid but surrounded by an  
outer core consisting of  
molten fluid iron & nickel. Hot inner  
core causes metals in outer core  
to flow, which generates electric  
currents, in turn inducing  
a magnetic field.

Good thing! This protects us from solar  
flares...

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Why are some materials magnetic?

To answer this, we need quantum physics, but we can discuss some basic aspects.

Two other ways to have magnetism in addition to moving charges

An electron has an intrinsic spin angular momentum (or just spin)

Associated w/ this is a magnetic dipole moment  $\vec{\mu}_s$  (recall this can come about from a current loop as well & indicates how it would align w/ a magnetic field)

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$\vec{\mu}_s$  &  $\vec{S}$  for an electron are related by

$$\vec{\mu}_s = -\frac{e}{m} \vec{S}$$

where  $e = 1.60 \times 10^{-19} \text{ C}$

&  $m = 9.11 \times 10^{-31} \text{ kg}$

1) The spin  $\vec{S}$  cannot be directly measured.

It is described by a quantum state which encodes quantum information. We can measure it along some axis.

2) If we measure along some axis, the outcomes are quantized, taking only certain values.

E.g., measuring in the  $Z$  direction

gives outcomes  $S_z = m_s \frac{h}{2\pi}$

where  $m_s = \pm 1/2$  &  $h$  is Planck's constant

(5)

outcome  $+1/2$  is called spin up

&  $-1/2$  is called spin down.

We can measure  $\vec{\mu}_s$  along z axis & get

$$\mu_{s,z} = -\frac{e}{m} \hbar S_z$$

$$= + \frac{eh}{4\pi m}$$

which correspond to being parallel or antiparallel w/ z axis

$$\frac{eh}{4\pi m}$$

is basic unit of spin magnetic moment of electron & other particles

& is called the

Bohr magneton.

when electron is placed in a magnetic field, there is an energy associated w/ orientation of it

(6)

$$U = -\vec{\mu}_s \cdot \vec{B}_{\text{ext}}$$

Another way to have magnetism:

when an electron is part of an atom, it has an orbital angular momentum

$\vec{L}_{\text{orb}}$  is associated to an orbital magnetic dipole moment  $\vec{\mu}_{\text{orb}}$

$$\vec{\mu}_{\text{orb}} = -\frac{e}{2m} \vec{L}_{\text{orb}}$$

Thus  $\vec{L}_{\text{orb}}$  can only be measured along an axis & has values

$$L_{\text{orb}, z} = m_l \hbar / 2\pi$$

where  $m_l = 0, \pm 1, \pm 2, \dots, \pm L$

(7)

when measuring, we get values

$$\mu_{\text{orb}} = -m_e \frac{eh}{4\pi m}$$

in terms of Bohr magneton  $\mu_B$

$$\mu_{\text{orb}} = -m_e \mu_B$$

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Three kinds of magnetism can occur in materials

- 1) Diamagnetism - in all materials but very weak. ~~They are~~ materials w/ only diamagnetism are "non-magnetic", including water, wood, & plastics. These materials respond by going away from field. Can lead to levitation effects, ...

2) Paramagnetism - atoms in material have a net magnetic dipole moment, but due to thermal fluctuations, they are randomly oriented.

In presence of field, the dipole moments align w/ field which then give the atoms a net magnetic field

(rare earth elements exhibit this behavior)

3) Ferromagnetic materials -

have permanent dipole moments

in atoms, giving a net dipole moment. These are the

"normal" magnets. iron, nickel, etc.

can encode information into these