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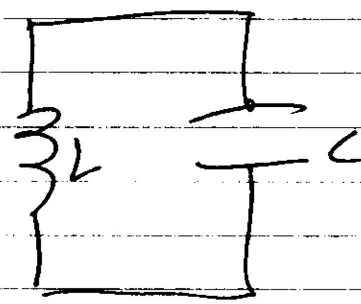
Lecture 32

12 Nov 2014

Last time talked about LC circuits

& how these transfer electric to magnetic energy & back as an oscillator



differential equation for circuit is

$$\frac{q(t)}{C} + L\frac{di(t)}{dt} = 0$$

$$\frac{di(t)}{dt} = \frac{d^2q(t)}{dt^2}$$

found solutions to be

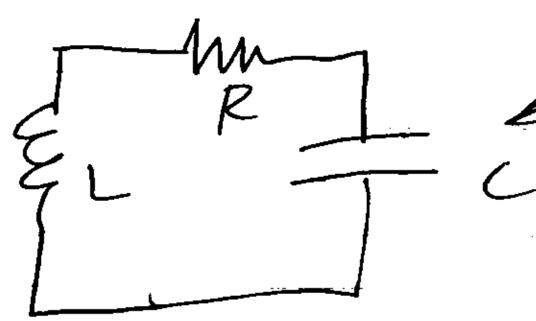
$$q(t) = Q\cos(\omega t + \phi)$$

where $\omega = \frac{1}{\sqrt{LC}}$

In reality, there is always some resistance, which leads to "damping" of oscillations (means that energy is decaying)

Before electric & magnetic energy go back & forth, but now this will be converted to thermal energy

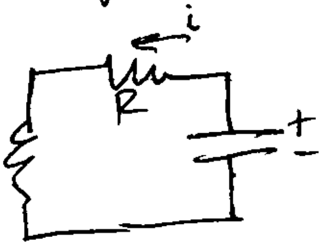
circuit we consider is RLC



some initial charge Q on capacitor

can solve the circuit in two ways:

QUESTION: loop rule



$$-\frac{q(t)}{C} - iR - L\frac{di(t)}{dt} = 0$$

$$\Rightarrow \frac{q(t)}{C} + \frac{dq(t)}{dt} R + L \frac{d^2 q(t)}{dt^2} = 0$$

solution to the diff. eq. is

$$q(t) = Q e^{-Rt/2L} \cos(\omega' t + \phi)$$

where $\omega' = \sqrt{\omega^2 - (R/2L)^2}$

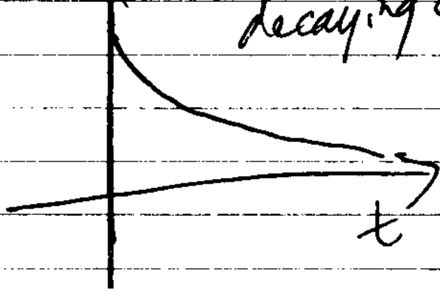
with $\omega = \dfrac{1}{\sqrt{LC}}$ (need R small enough)

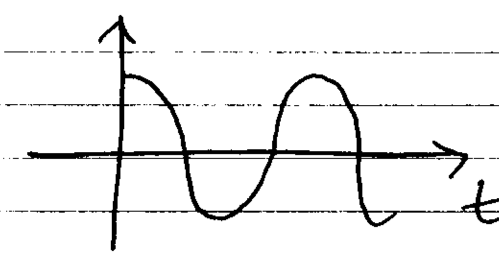
check this solution yourself...

product of two terms

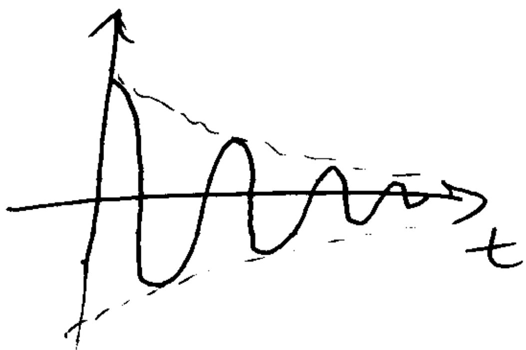
$Q e^{-Rt/2L}$ & $\cos(\omega' t + \phi)$

decaying exponential





product then looks like



damped oscillations

Another way to get circuit equation:
use energy

Magnetic energy of inductor + electrical energy of cap. is

$$\frac{1}{2}Li^2 + \frac{1}{2C}q^2 = U$$

rate at which this decreases is equal to power delivered to resistor in form of thermal energy

$$\Rightarrow \quad -\frac{dU}{dt} = i^2 R$$

$$\Rightarrow \quad -\frac{d}{dt}\left\{\frac{1}{2}Li^2 + \frac{1}{2C}q^2\right\} = i^2 R$$

$$\Rightarrow$$

$$-\frac{1}{2}L \cdot 2i \cdot \frac{di}{dt} - \frac{1}{2C} \cdot 2q\,\frac{dq}{dt} = i^2 R$$

$$\Rightarrow \quad -L\frac{di}{dt} - \frac{q}{C} = iR$$

same equation

Alternating current -

oscillates back & forth w/ time

- much more practical for the distribution of electrical power over large distances

Why? For fixed power $P = VI$

can make voltage high or current high.

If current is high, then much more power lost to heat $i^2 R$

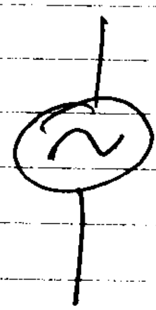
But if voltage high, then less is lost to heat V^2/R

Also it's easier to convert AC from one voltage to another

(but it was more expensive & inefficient to do so w/ DC)

↳ and AC generators are cheaper & more efficient

Model an AC generator as



w/ EMF

$$\mathcal{E} = \mathcal{E}_m \sin \omega_d t$$

max EMF

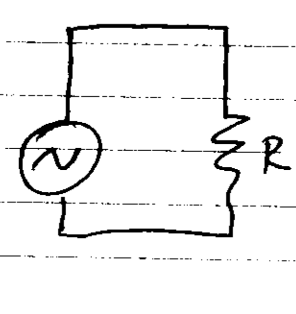
frequency of oscillation

current is then

$$i = I \sin(\omega_d t - \phi)$$

phase ϕ is there b/c current does not have to be in phase w/ EMF

Three circuits to consider:



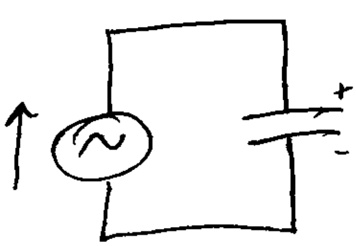
QUESTION:
What is current through resistor if $\mathcal{E} = \mathcal{E}_m \sin(\omega_d t)$?

$\Rightarrow$ current $i = \dfrac{\mathcal{E}_m}{R} \sin(\omega_d t)$

current is in phase w/ AC EMF for purely resistive load.

Checkpoint:
If we increase frequency, does amplitude of voltage + current change?

Next circuit:



What is current?
use loop rule to get

$$\mathcal{E}_m \sin(\omega_d t) = \frac{q(t)}{C}$$

$$\Rightarrow q(t) = C\mathcal{E}_m \sin(\omega_d t)$$

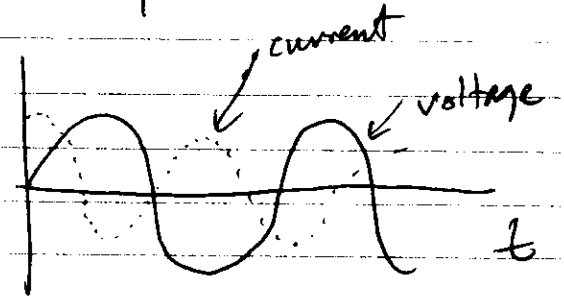
$\Rightarrow$ current $i(t) = \dfrac{dq(t)}{dt} = \omega_d C \mathcal{E}_m \cos(\omega_d t)$

$= \omega_d C \mathcal{E}_m \sin(\omega_d t + 90^\circ)$

$\Rightarrow \phi = -90$

so current & voltage are out of phase by 90°

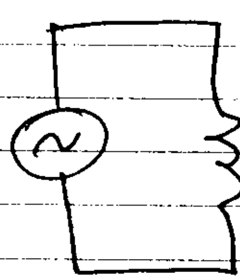
plot



interpretation: voltage follows current b/c hard to build up a potential difference, current goes immediately as device gets charged

Inductive load

use loop rule



$\mathcal{E}_m \sin(\omega_d t)$ to get voltage across inductor

current we get from

$$\mathcal{E}_m \sin(\omega_d t) = L \frac{di}{dt}$$

integrate to get

$$\mathcal{E}_m \int \sin(\omega_d t) = L i$$

$$\Rightarrow \quad -\frac{\mathcal{E}_m}{\omega} \cos(\omega_d t) = L i$$

$$\Rightarrow \quad i(t) = -\frac{\mathcal{E}_m}{\omega L} \cos(\omega_d t)$$

$$= \frac{\mathcal{E}_m}{\omega L} \sin(\omega_d t - 90^\circ)$$

$\Rightarrow$ current follows voltage
b/c inductor opposes changes in current