Lecture 29

5 NOV 2014

Faraday Law of Induction:

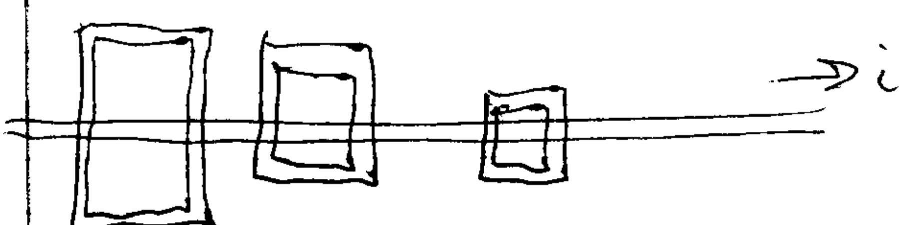
Magnitude of EMF induced in a conducting loop is equal to rate of change of magnetic flux through that loop:

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

where $\Phi_B = \int \vec{B} \cdot d\vec{A}$

surface integral over conducting loop

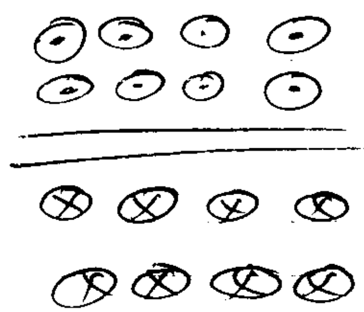
can use this to attack homework question:



If current i is constant,
what is induced current
for each conducting loop?

will be proportional to
induced EMF

use 2^{nd} right hand rule to
get B-field is



& magnitude is
$$B = \frac{\mu_0 i}{2\pi r}$$

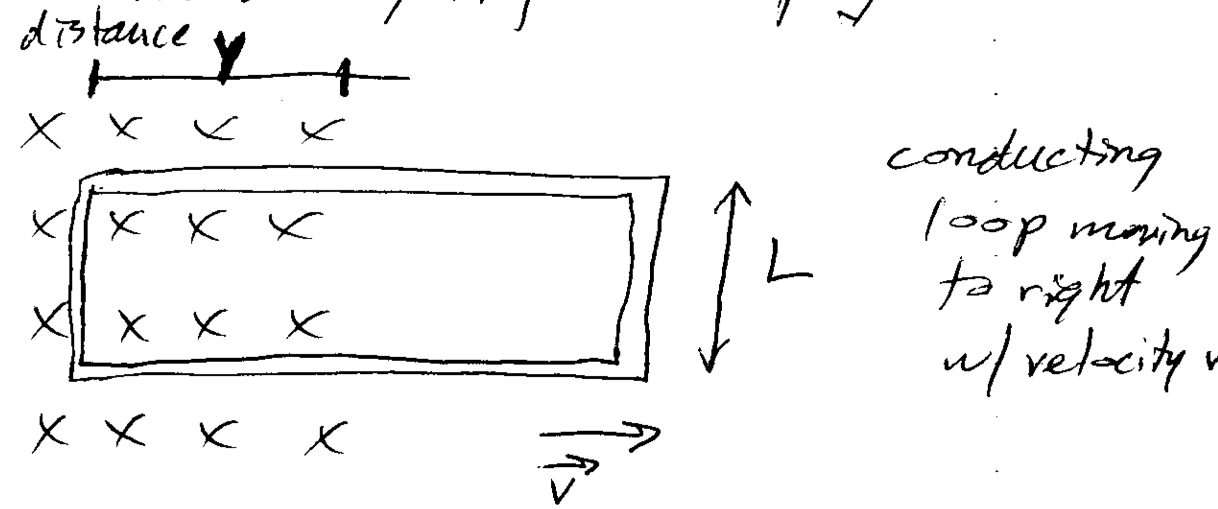
Since current i is constant,
B-field not changing w/ time
& neither is flux, so
all have zero induced
current.

What if current i is increasing?

Then B-field gets more intense over time, but flux for the symmetric ones (1 & 3) is equal to zero at all times.
Non-symmetric one has increasing flux & so non-negligible induced EMF & current.

Induction & Energy Transfer

Given B a uniform magnetic field going into page



To pull the loop to the right, you need to apply a force & since work $W = F \cdot d$, the power $P = \frac{dW}{dt} = Fv$

where v is velocity

Moving loop to the right decreases area which encloses magnetic field.

<u>QUESTION:</u> What is magnetic flux for the snapshot given?

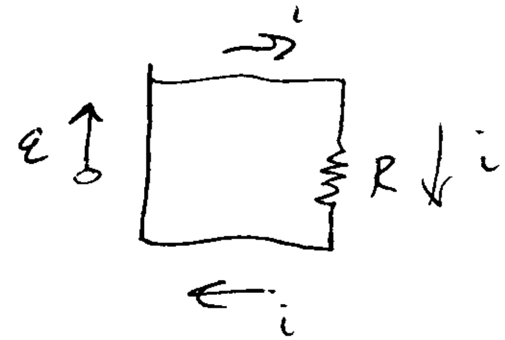
$$\Phi_B = BA = B \cdot L \cdot y$$

From Faraday law, we get magnitude of induced EMF is

$$\mathcal{E} = \frac{d\Phi_B}{dt} = \frac{d}{dt}(BLy) = BL\frac{dy}{dt}$$

$= BLv$

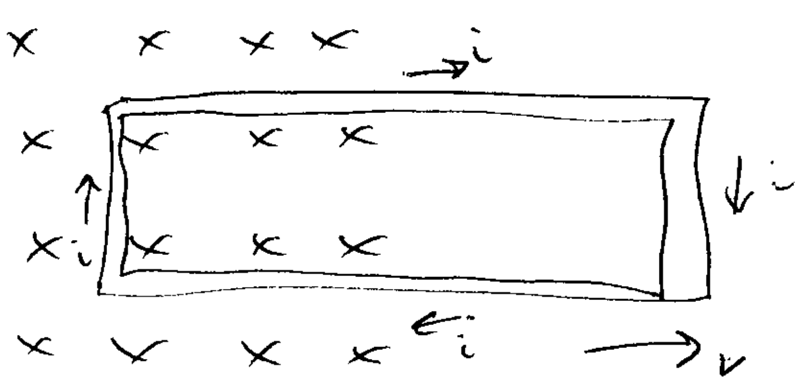
We can write an effective circuit diagram for the loop (which has some resistance) as



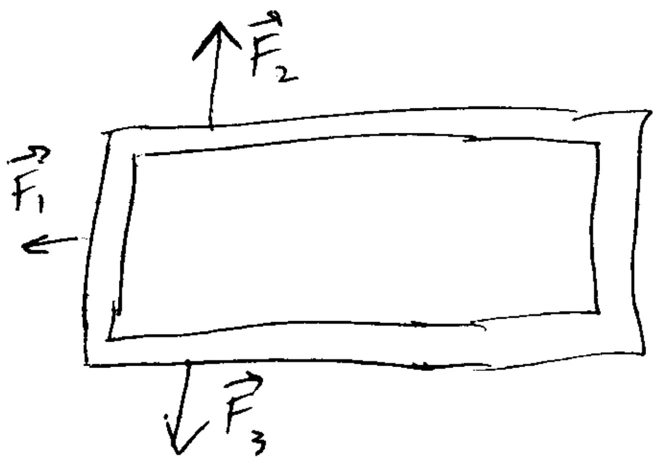
So the induced current is given by

$$i = \frac{\mathcal{E}}{R} = \frac{BLv}{R}$$

Back to original picture:



Since there is an induced current, there will be a force on the three segments near magnetic field



use

$$\vec{F} = i\vec{L} \times \vec{B}$$

F_2 & F_3 cancel b/c in opposite directions & have same magnitude

$\vec{F}_1$ given by

$$\vec{F}_1 = i\left(\vec{L} \times \vec{B}\right)$$

& magnitude of force is

$$F_1 = iLB$$

Substituting for i, we get, $F_1 = \frac{B^2L^2v}{R}$

Power is given by

$$P = Fv = \frac{B^2 L^2 v^2}{R}$$

power dissipated as thermal energy in conducting loop is

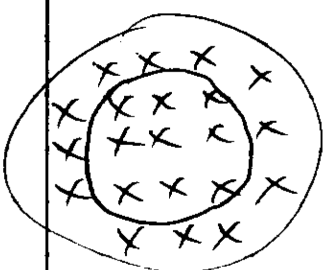
$$P = i^2 R = \left(\frac{BLv}{R}\right)^2 R$$

$$= \frac{B^2 L^2 v^2}{R}$$

$\Rightarrow$ work you do to move loop is dissipated as thermal energy in loop.

Induced Electric Fields

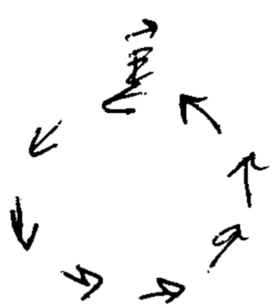
Suppose cylinder containing copper ring w/ uniform magnetic field increasing



Increasing Magnetic flux through ring induces current & as such, induces electric field.

current around ring

E-field is



So a changing magnetic field produces an electric field.

But this actually happens independent of whether there is a copper ring

Suppose a test charge q_0 is moving around a circular path of radius r.

Work done on it in one revolution is $W = q_0 \mathcal{E}$
where $\mathcal{E}$ is induced EMF

But we also have that

$W = q_0 \oint \vec{E} \cdot d\vec{s}$ where $\vec{E}$ is E-field along circular path (closed)

So this means that

$\mathcal{E} = \oint \vec{E} \cdot d\vec{s}$ and we can rewrite the Faraday law as

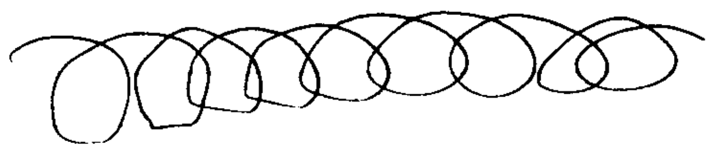
$$\oint \vec{E} \cdot d\vec{s} = \frac{-d\Phi_B}{dt}$$

equivalent to saying that a changing B-field produces an E-field

Inductors & Inductance

An inductor can be used to produce a desired magnetic field

solenoid



establishing a current i through it, the current produces a magnetic flux Φ_B

Inductance of this is defined as

$$L = \frac{N \Phi_B}{i}$$

where N is # of turns