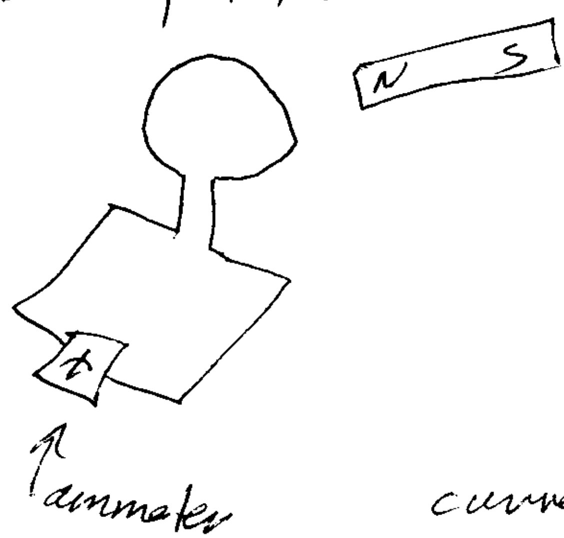
# Lecture 27

3 Nov 2014

## Ch. 30 - Induction

Magnetic field can produce an electric field that can drive a current.

2 experiments to prepare for induction




1) move a bar magnet towards loop.
2) current appears in the circuit

current will stop when movement stops.

move the bar magnet away & current starts again but in the opposite direction

Conclusions from experiment one

1) Current appears if there is relative motion between loop + magnet

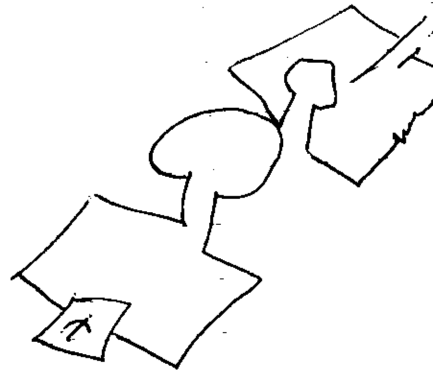
2) Faster motion produces greater current.

3) If moving magnet's north pole towards loop causes clockwise current, then moving it away causes counter clockwise current.

process of producing current + EMF is called <u>induction</u>.

<u>2nd experiment:</u>

current is induced in circuit w/o battery when turning on + off circuit w/ battery but not when there is constant current



$\vec{dA}$ is a vector of magnitude $dA$ perpendicular to a differential area.

Special case: QUESTION

uniform magnetic field perpendicular to loop

Then $\vec{B} \cdot \vec{dA} = B\,dA$

$$\& \ \Phi_B = \int \vec{B} \cdot \vec{dA} = \int B\,dA$$

$$= B \int dA$$

$$= BA$$

SI unit for magnetic flux is

1 Weber = 1 Tesla - $m^2$
(Wb)

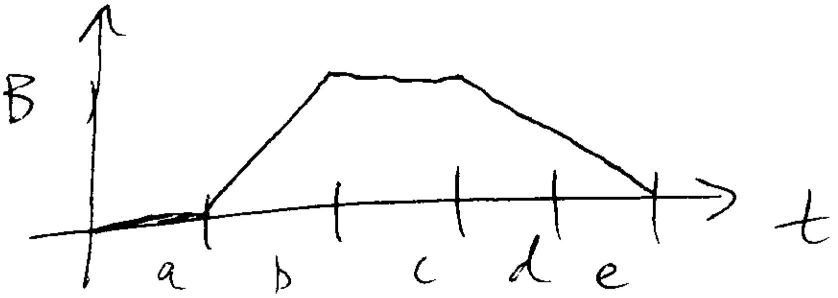
Can state Faraday's law

Magnitude of induced EMF in a loop is equal to rate of change of magnetic flux

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

(minus sign ⇒ induced EMF acts to oppose change in magnetic flux)

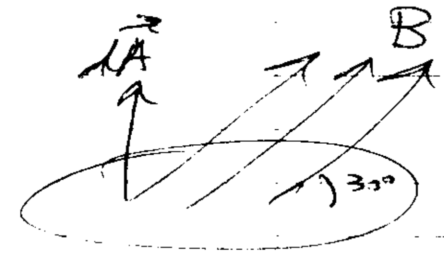
QUESTION: graph gives magnitude $B(t)$ of a uniform magnetic field that exists throughout a conducting loop, w/ direction of field perpendicular to the plane of the loop. Rank regions according to magnitude of EMF induced in the loop




$b > d = e > c = a$

Example: Closed loop of wire encloses an area of $A = 1 m^2$ in which a uniform magnetic field exists at 30° to plane of the loop. The magnetic field is decreasing at a rate of $\frac{dB}{dt} = 1\ T/s$ resistance of wire is $10\ \Omega$. What is induced current?

Use Faraday's law.




$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

$$= \int B\, dA \cos(60°)$$

$$= BA\cos(60°) = \frac{BA}{2}$$

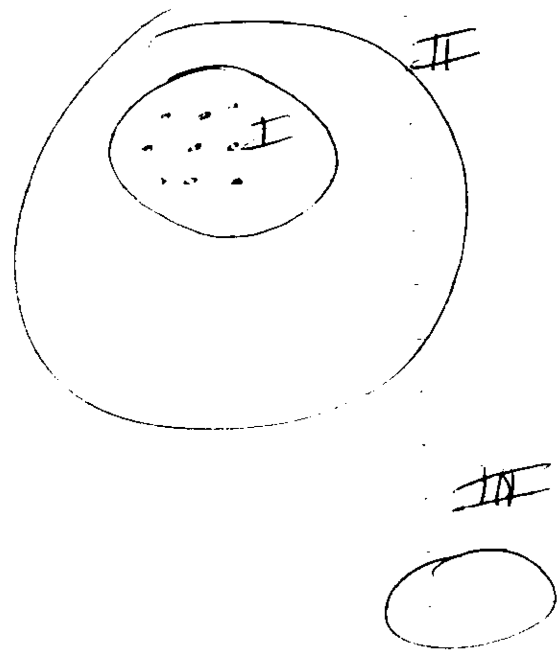
$$\Rightarrow |\mathcal{E}| = \frac{d\Phi_B}{dt} = \frac{A}{2}\frac{dB}{dt} \Rightarrow i = \frac{|\mathcal{E}|}{R} = \frac{A}{2R}\frac{dB}{dt}$$

$$= \frac{(1 m^2)}{2(10\Omega)}(1 T/s) = .05 A$$

## QUESTION:

### Another example:

B=0 everywhere except in region I where B is uniform pointing out of the page & increasing at a steady rate




rank the 3 loops in order of increasing induced EMF

I = II > III

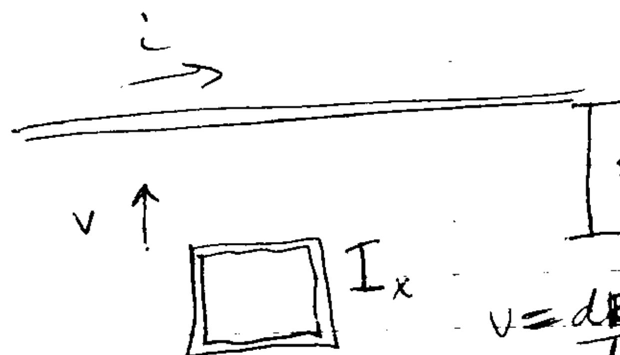
↑ same flux

↑ no flux

Another example:

- infinitely long wire carries a current $i$
- square loop w/ side length $L$ moving towards wire w/ constant velocity $v$

What is the EMF induced in the loop?




$v = \frac{dR}{dt}$

First get the flux through it.

we showed before that B-field @ distance $r$ from a wire is

$$B = \frac{\mu_0 i}{2\pi r}$$

So pick a strip of width $dx$

B-field @ every point along strip is $\frac{\mu_0 i}{2\pi(R+x)}$ + area of strip is $L\,dx$

$\Rightarrow$ flux is

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

B-field is aligned w/ normal vector for loop

$$\Rightarrow \quad \Phi_B = \int_0^L \frac{\mu_0 i}{2\pi (R+x)} L\,dx$$

$$= \frac{\mu_0 i L}{2\pi} \log (R+x) \Big|_0^L$$

$$= \frac{\mu_0 i L}{2\pi} \log \left[ \frac{R+L}{R} \right]$$

Faraday law of induction

$$\Rightarrow \quad \mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{\mu_0 i L}{2\pi} \frac{d}{dt}\left(\log\left(1+\frac{L}{R}\right)\right)$$

$$= \frac{\mu_0 i L}{2\pi} \left[ \frac{R}{R+L} \right] \frac{L}{R^2} \frac{dR}{dt}$$

$$= \frac{\mu_0 i L^2}{\ldots} v \cdot \frac{1}{\ldots}$$