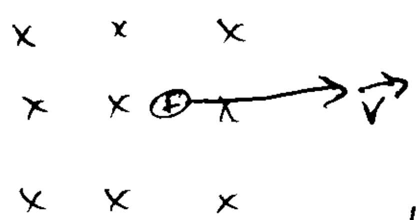
Lecture 26

29 OCT 2014

Review of 2 Right-hand rules

$$\vec{F}_B = q(\vec{v} \times \vec{B})$$

0) find plane formed by $\vec{v}$ & $\vec{B}$



1) start w/ index finger pointing in direction of $\vec{v}$

2) end up w/ finger in direction of $\vec{B}$

3) direction of thumb is direction of force on a positive charge

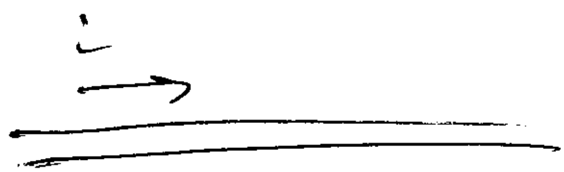
Other right hand rule

What is B-field induced by moving charge? Use Biot-Savart law

$$\vec{B} = \frac{\mu_0}{4\pi r^2} q \, \vec{v} \times \hat{r}$$

other version

$$d\vec{B} = \frac{\mu_0 i}{4\pi r^2} d\vec{\ell} \times \hat{r}$$

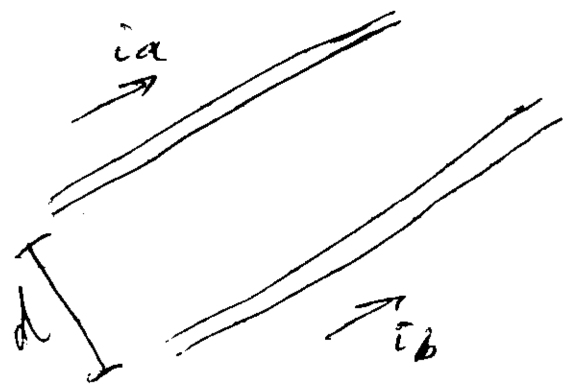


1) point right thumb in direction of current

2) curl fingers around

3) direction of fingers indicates direction of magnetic field

Force between 2 parallel currents



QUESTION:

What is magnetic field direction at wire b due to current i_a?

$\vec{B}_a$ down ↓

What is magnitude?

last time showed

$$B_a = \frac{\mu_0 i_a}{2\pi d}$$

Now if there is a magnetic field going down & current i_b, there will be a force on wire b given by Lorentz force law

$$\vec{F}_{ba} = i_b \vec{L} \times \vec{B}_a$$

force is towards wire a

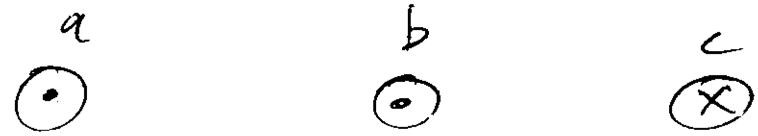
$$\Rightarrow \quad F_{ba} = (i_b L) \frac{\mu_0 i_a}{2\pi d}$$

QUESTION: What is direction of force if the currents go in opposite directions?

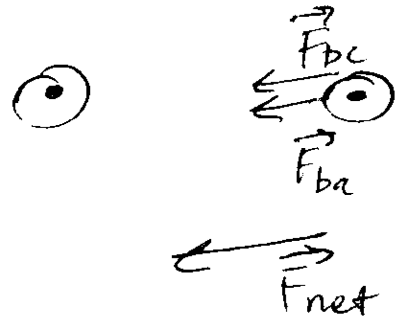
The wires will repel each other

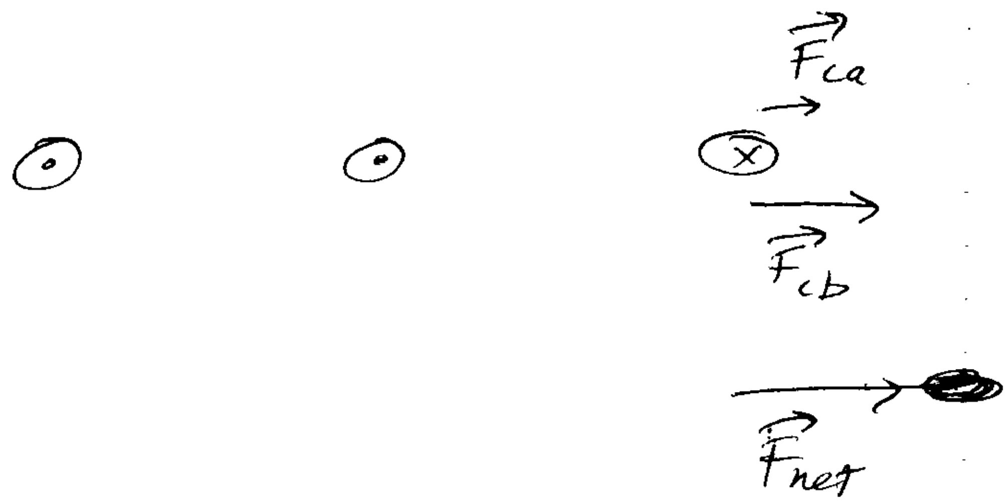
QUESTION: Suppose there are three straight parallel wires each carrying same current.

Rank wires according to magnitude of force on each due to current in the others.



remember that currents in the same direction attract, while opposites repel.



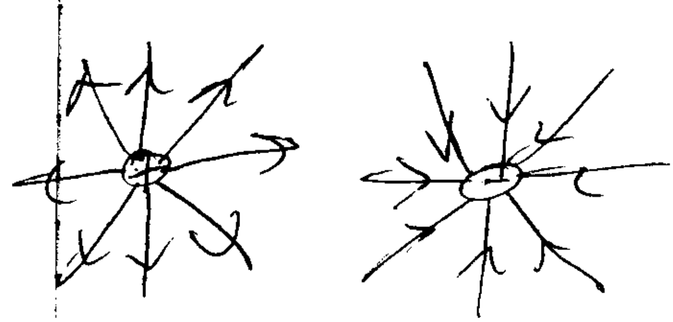


So $b > c > a$

Review: Gauss' law for E-fields

flux $\propto$ net enclosed charge

i.e. $$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$



sources + sinks for charge

Gauss' law for B-fields

remember that B-fields have no sources or sinks.

So the number of field lines going into any Gaussian surface is equal to the number going out

$$\Rightarrow \quad \oint \vec{B} \cdot d\vec{A} = 0$$

magnetic flux is always equal to zero.

This is one of the four Maxwell's equations.

Ampere's law can be derived from the Biot-Savart law.

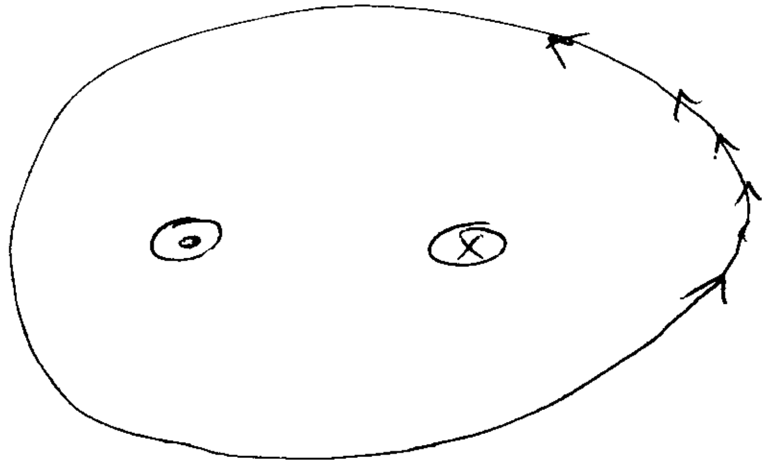
We will not do this derivation but merely state Ampere's law as

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \, i_{enc}$$

What does this mean?

Suppose currents going in & out of the board

We can figure out a relation between currents & magnetic field by applying Ampere's law.

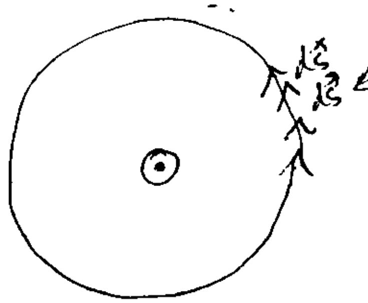


take a closed loop around currents

for every point on loop, we have $\vec{B}$ & $d\vec{s}$. Then compute $\vec{B} \cdot d\vec{s}$ & summing over the whole loop gives $\oint \vec{B} \cdot d\vec{s}$. The Ampere law says that this is equal to the sum of the enclosed currents. ($\times \mu_0$)

Current counted positive if when curling fingers of right hand around loop thumb points in direction of current & otherwise negative

Can use Ampere's law to revisit our earlier calculation



enclosed current is i

so $\mu_0 i = \oint \vec{B} \cdot d\vec{s}$

But B-field direction is the same as direction for $d\vec{s}$

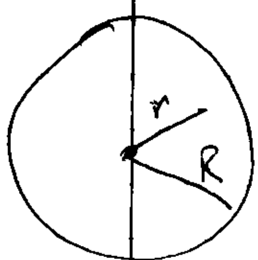
so $\oint \vec{B} \cdot d\vec{s} = \oint B\,ds$

Furthermore B is constant b/c distance is the same for a circle

$= B \oint ds$

$= B\, 2\pi r$ (and we get earlier result)

Current + B-field inside a wire



What is B-field @ a distance r from center? $(r \leq R)$

Use Ampere's law.

$i_{enc} = J(\pi r^2)$

↑ current density

but J for wire when current is distributed uniformly is

$J = \frac{i}{\pi R^2}$

area of cross section

So $i_{enc} = \frac{i \pi r^2}{\pi R^2}$

+ then $\mu_0 i_{enc} = \oint \vec{B} \cdot d\vec{s}$

But same calculations give

$= B \cdot 2\pi r$

$\Rightarrow B = \frac{\mu_0 i r}{2\pi R^2}$

Plot of B-field inside and outside wire: