Lecture 25

27 OCT 2014

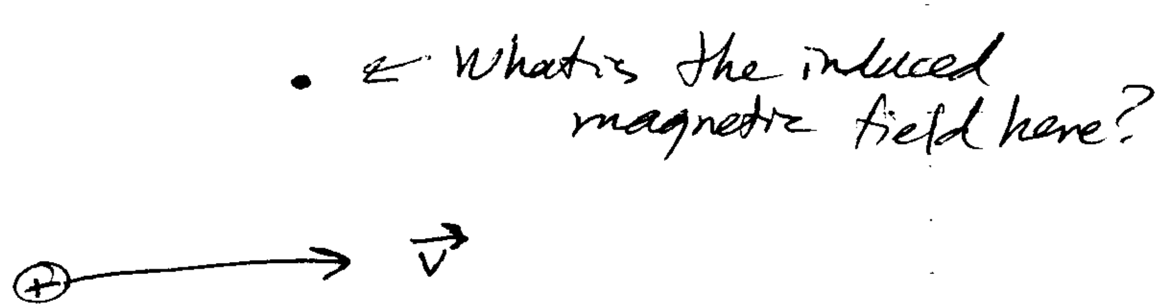
Last week, we talked about the Lorentz force law extensively

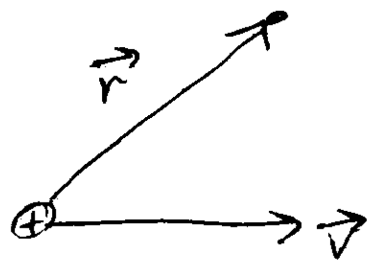
$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

Force exerted on a moving charge

It turns out that a moving charge induces a magnetic field in the space around it. (Experiment of Oersted)

So consider a moving charge





$\vec{r}$ is a vector from the moving point charge to the location of interest. It is found that

$$\vec{B} = \frac{\mu_o}{4\pi r^2} q \, \vec{v} \times \hat{r}$$

where $\hat{r}$ is a unit vector in direction of location & r^2 is magnitude of distance

So an inverse square law!

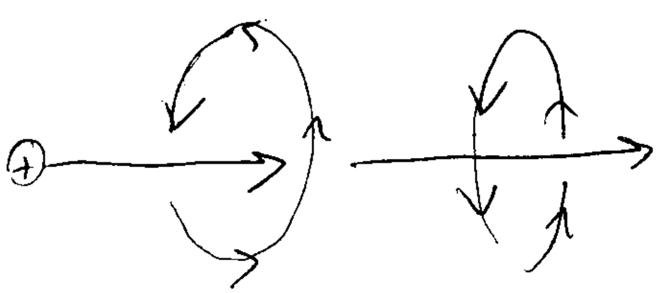
$$\mu_o = 4\pi \times 10^{-7} \ \frac{V \cdot s}{A \cdot m}$$

so B-field comes out of the board

What about the B-field
in this scenario?

going into board

can draw a picture like

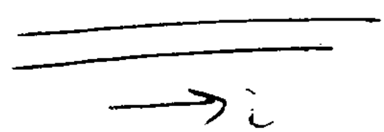


moving charge induces a circular
magnetic field in the space around it

another right-hand rule:
point thumb in velocity direction &
curl your fingers around to get
directions of magnetic field

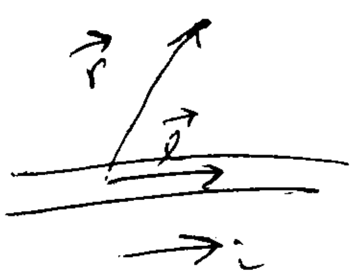
Biot - Savart law is
equivalent to this (similar manipulation to last week's)

given a current flowing through a straight wire



we can define a length vector
$\vec{\ell}$ as before &
use that $q\vec{v} = i\vec{\ell}$ to
find

$$\vec{B} = \frac{\mu_0}{4\pi r^2} \, i\,\vec{\ell} \times \hat{r}$$



so the field in some sense loops
around the wire.

Infinitesmal version of the law

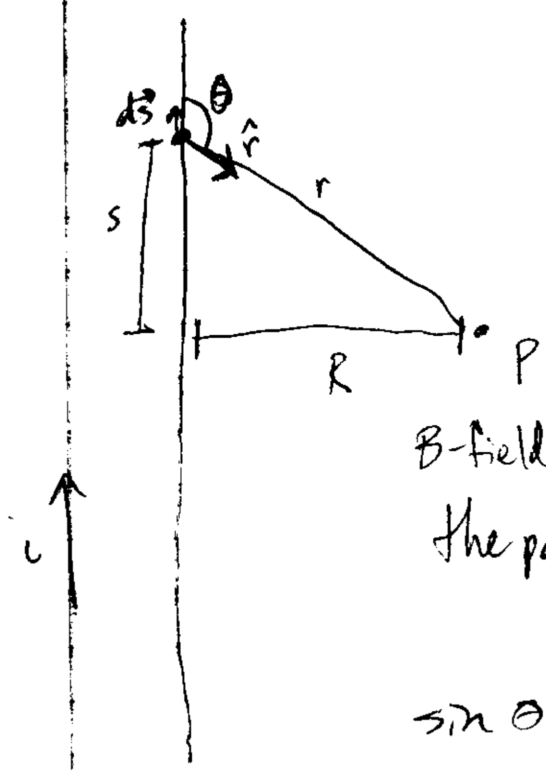
$$d\vec{B} = \frac{\mu_0}{4\pi r^2} i \, d\vec{l} \times \hat{r}$$

Apply this + superposition principle

Example:

calculating the magnetic field due to a current flowing through an infinitely long wire

Idea is to sum up all of the contributions



$$dB = \frac{\mu_0}{4\pi r^2} i \, ds \sin\theta$$

B-field going into the page

get everything in terms of s & R

$$r = \sqrt{s^2 + R^2}$$

$$\sin\theta = \sin(\pi - \theta) = \frac{R}{\sqrt{s^2+R^2}}$$

So $B = \int dB$

observe symmetry, so it suffices to integrate from 0 to ∞ & double the result

$$2\int_0^{\infty} \frac{\mu_0 i \, R}{4\pi \left(s^2+R^2\right)^{3/2}} \, ds$$

$$= \frac{\mu_0 i}{2\pi R}\left[\frac{s}{\left(s^2+R^2\right)^{1/2}}\right]_0^{\infty}$$

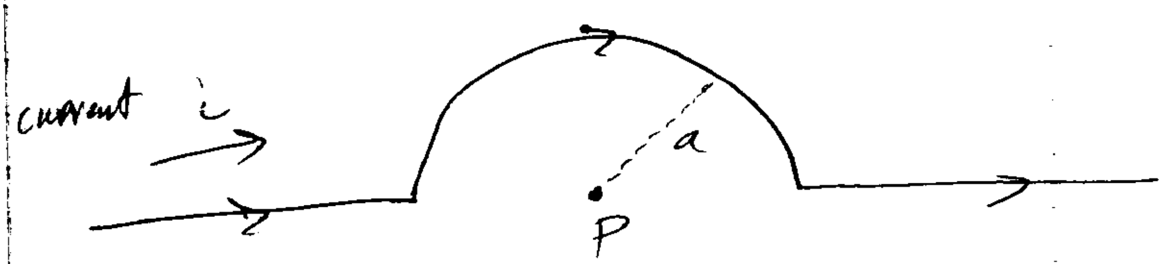
$$= \frac{\mu_0 i}{2\pi R}$$

~~For a semi-infinite wire~~

QUESTION: What would we get for a semiinfinite straight wire?

<u>Another example</u>:

Current flowing through wire:

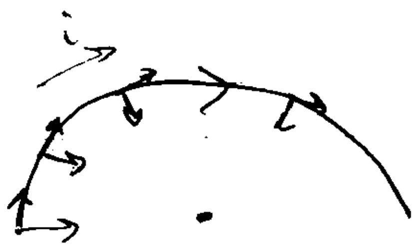


What is magnetic field at point P?

<u>QUESTION</u>: What is its direction?

How to calculate it?

1) straight portions don't contribute anything b/c the cross product of length vectors + distance vectors is zero since they are parallel
2) so focus on the circle



$$dB = \frac{\mu_0 \, i \, dl \sin\theta}{4\pi r^2}$$

θ is always 90° $\Rightarrow$ $\sin\theta = 1$

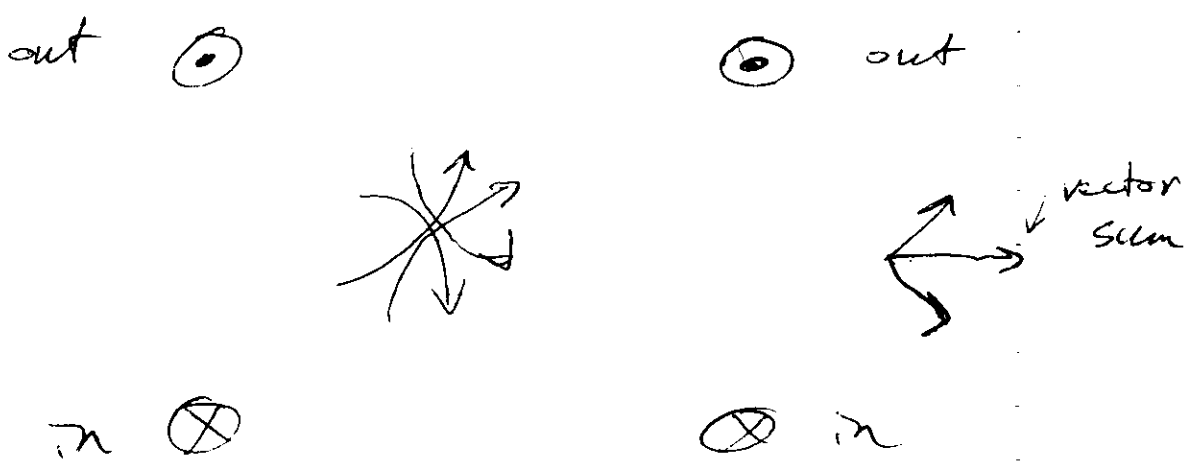
r is constant & equal to a

So
$$\int dB = \frac{\mu_0 i}{4\pi a^2} \underbrace{\int dl}_{\frac{1}{2} \text{ just circumference } = \pi a}$$

$$= \frac{\mu_0 i}{4a}$$

QUESTION:

Suppose two currents going-in
+ two going out (all equal magnitude)



What is the direction
of B at the center?

going to the
right