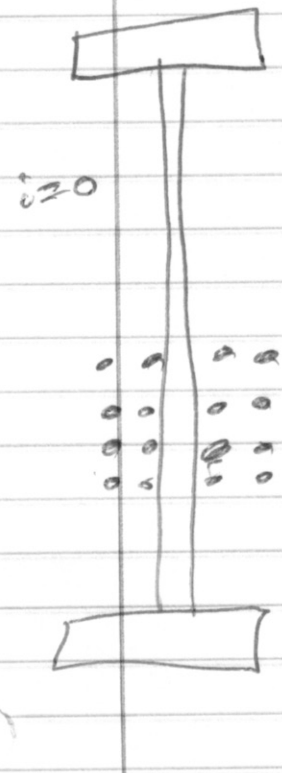
Lecture 24

24 Oct 2014

Hall effect – magnetic field exerts a force on electrons moving through a wire.

Suppose that we have a flexible wire + situation is

then we turn on current going ↑.

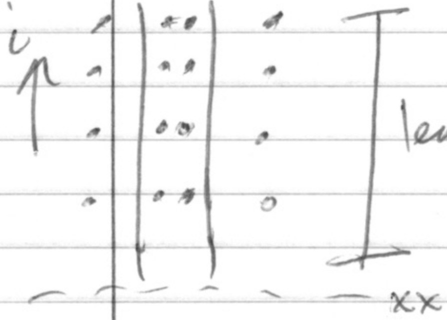
QUESTION: Which way will [illegible] force be?

force goes to the right

Wire actually moves to the right

reversing the direction of the current then reverses the direction of the force.

Let's zoom in on the wire



conduction electrons in length L this part pass through plane xx in time

$$t = L/v_d$$

⇒ that the charge going by in this time is $q = it = iL/v_d$

We know that the magnitude of the magnetic force is

$$F_B = q\, v_d B \sin\varphi$$

where φ is the angle between $\vec{v}_d$ & $\vec{B}$

so we get

$$F_B = i \frac{L}{v_d} v_d B \sin\varphi$$

$$= i L B \sin\varphi$$

The vector equation is then given by

$$\vec{F}_B = i\, \vec{L} \times \vec{B}$$

where $\vec{L}$ is a length vector w/ magnitude L & going in direction of this equation is equivalent to

$$\vec{F}_B = q\,\vec{v} \times \vec{B} \quad \&$$

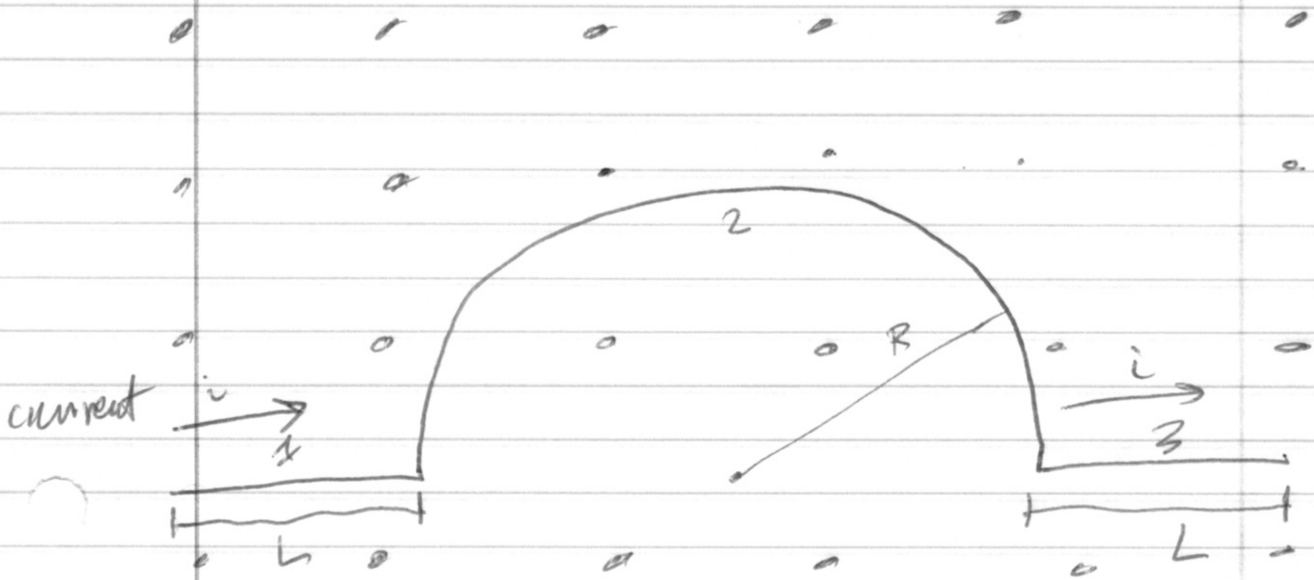
can be used as a good substitute in practice since it is easier to measure magnetic force acting on a wire than on a single charge.

If the wire is not straight or ~~the~~ field is not uniform, then we need to use a differential form of this law to get the total force:

$$d\vec{F}_B = i\,d\vec{L} \times \vec{B}$$

Example:

uniform field



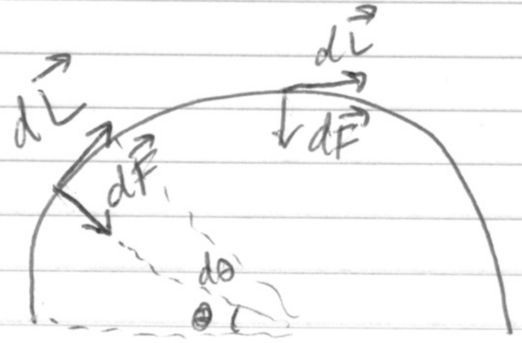
What is the total force on the wire?

QUESTION: What is the force on the 1st & 3rd parts?

$F_1 = F_3 = iLB$

$F_1 \downarrow \quad F_3 \downarrow$

to get for the middle one, we can use calculus



$d\vec{L}$ can be taken to have magnitude $dL = R\,d\theta$

so that it is a little bit of arclength along the semicircle.

Applying law, $dF = i B\,dL = i B R\,d\theta$

the x components of the forces will cancel out (ones on left and right)

So we just need to keep track of y components.

y component is $dF \sin\theta$

So we add up all the y components

$$F_2 = \int dF_y = \int_0^{\pi} \sin\theta\, dF = iBR\int_0^{\pi} \sin\theta\, d\theta$$

$$= 2iRB$$

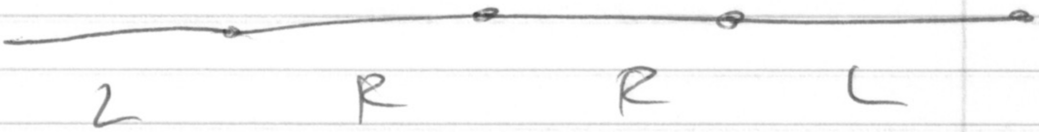
So total force is

$$F_1 + F_2 + F_3$$

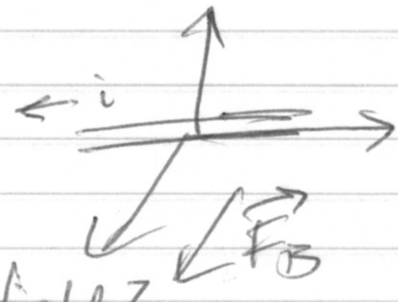
$$= 2iLB + 2iBR$$

$$= 2iB(L+R)$$

This force is the same as that for a straight wire



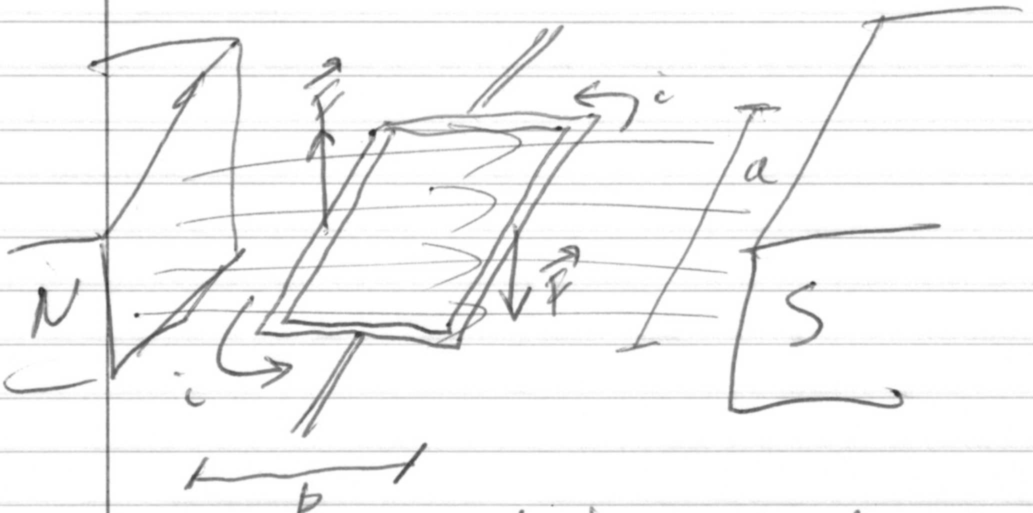
QUESTION:



Which direction is field?
down

Torque on a current loop

Imagine the following



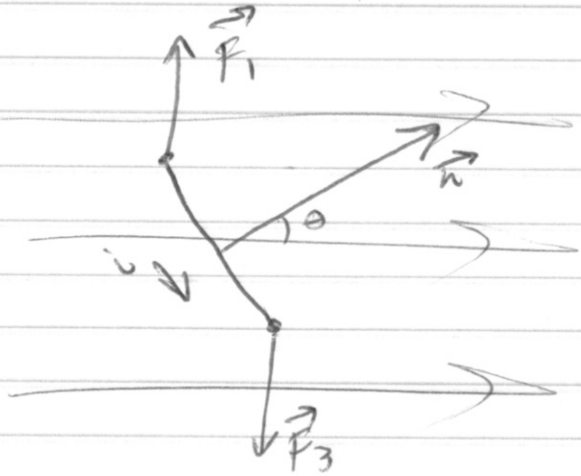
current loop inside magnetic field.

magnetic field & current will cause magnetic forces that make a torque. (tendency to twist)

magnitude of each force is ~~[illegible]~~ iaB

~~when @ a different angle~~

for a rotated version + different perspective



Moment arm for $\vec{F}_1$ is $b/2 \sin\theta$

same for $\vec{F}_3$ (similarly, projection of $\vec{F}_1$ onto $\vec{n}$ is $\vec{F}\cos(90-\theta) = \vec{F}\sin\theta$)

$\Rightarrow$ total torque is $iaB\,(b\sin\theta)$

$$= i\,ab\,B\sin\theta$$

$$= i\,A\,B\sin\theta$$

where A is area of loop

torque will act to align the normal vector w/ the field.

If the coil has N loops then torque is $N\,(i\,A\,B\sin\theta)$

Magnetic Dipole Moment of

coil is $\mu = NiA$

Then $\tau = \mu B \sin\theta$ &

vector equation is

$\vec{\tau} = \vec{\mu} \times \vec{B}$ where $\vec{\mu}$ points in direction of normal vector

(very similar to equation for electric dipole moment)

Can assign a potential energy for various configurations

$$U(\theta) = -\vec{\mu} \cdot \vec{B}$$

same as for electric dipole