Lecture 12

21 SEP 2014

electrostatic force is a conservative force - work done in moving a particle from one place to another is independent of the path taken

- the same is true for the gravitational force.

⇒ we can assign a potential to every point in space, & force changes the potential energy by moving a particle from one location to another & change in P.E. is independent of path.

A system of two or more charged particles has electric potential energy.

Change in potential energy is given by

$$\Delta U = U_f - U_i = -W$$

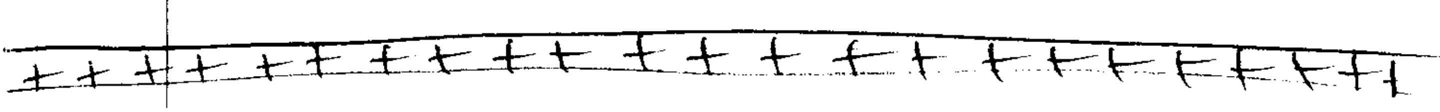
where U_f is final potential energy &

U_i " initial " "

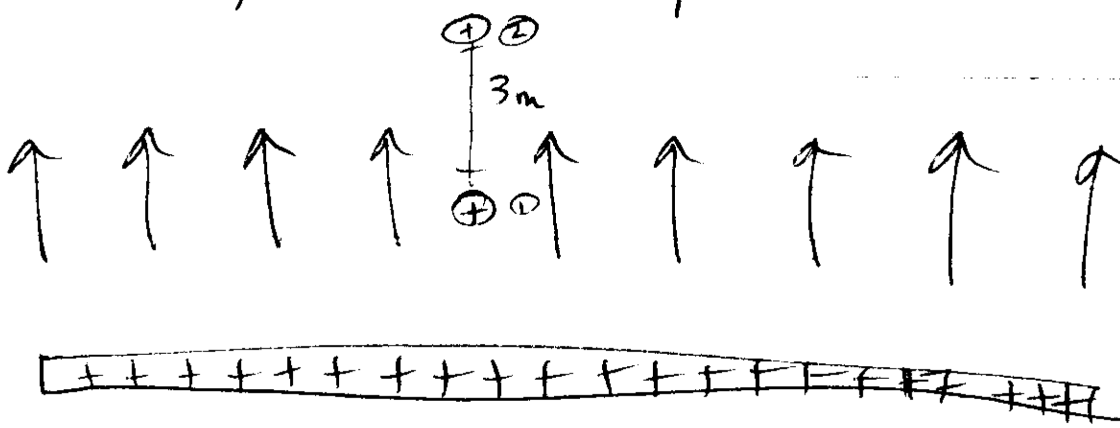
& W is work done by electric force on particles

Example: Suppose a uniform σ sheet of charge w/ surface charge density σ

What is E-field at any point above sheet? $E = \frac{\sigma}{2\varepsilon_0}$



What is change in potential energy for a positive charge of 2C to go 3m up?



Work = Force · distance

$$F = q \cdot E = 2C \cdot \frac{\sigma}{\varepsilon_0} \frac{N}{C}$$

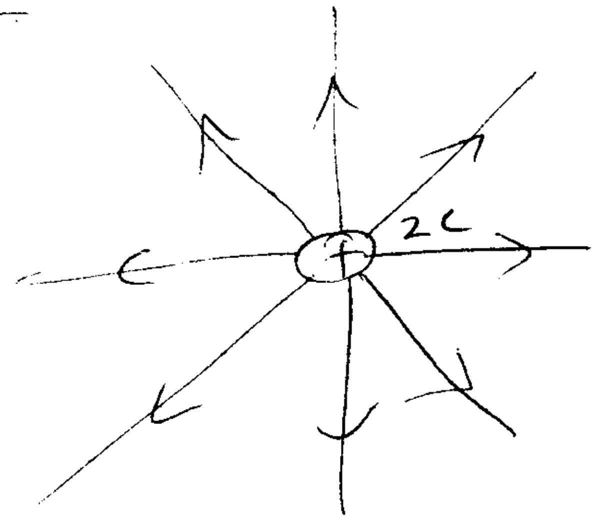
$$= \frac{2\sigma}{\varepsilon_0} N$$

$$F \cdot d = \frac{2\sigma \cdot 3}{\varepsilon_0} N \cdot m = \frac{6\sigma}{\varepsilon_0} N \cdot m \ (J)$$

potential energy goes down, so E-field does work on particle ~~negative work needed~~ ~~direction of field~~.

QUESTION: What about for an electron?

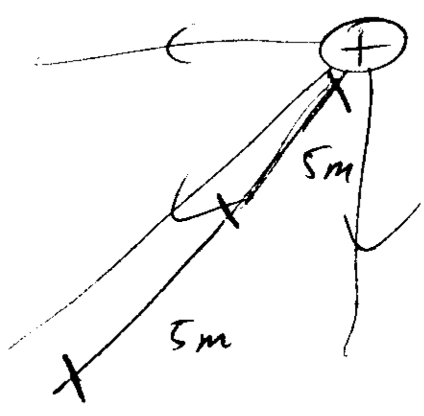
Another example:



What is the work required to move a ⊕ of 3C at a distance r towards the ⊕ charge through a distance of 5m when starting 10m away?

here we need calculus ...

divide up the interval into little segments of length dr & over these little distances the force will be constant & given by Coulomb law



$$F = \frac{k \cdot q_1 \cdot q_2}{r^2}$$

So the work for a little interval is equal to $F \cdot dr$

$$= \frac{k \cdot q_1 \cdot q_2}{r^2} \cdot dr$$

we then need to ~~integrate~~ sum over all of these little distances (integrate)

$$\int_{10}^{5} \frac{k \cdot q_1 \cdot q_2}{r^2} \, dr = k \cdot q_1 \cdot q_2 \int_{10}^{5} \frac{1}{r^2} dr$$

$$= k \cdot q_1 \cdot q_2 \left[-r^{-1} \Big|_{10}^{5} \right]$$

$$= k \cdot 2C \cdot 3C \left[-\frac{1}{5} + \frac{1}{10} \right]$$

$$= -\frac{k \cdot 6 C^2}{10 \text{ m}}$$

We need to go against direction of E-field so work put into system is

$\frac{k \cdot 6 C^2}{10 \text{m}}$ + difference of potential energies is the same amount potential energy is really about differences in PE of various locations. We need some reference point in order to establish what is the zero of potential energy.

convention is for the zero of potential energy to be when particles have an ∞ separation between them

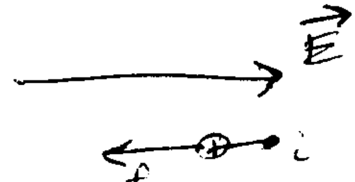
From original definition,

$$\Delta U = U_f - U_i = -W$$

So $U_i = 0$ @ ∞ $\Rightarrow$

$$U_f = U = -W_\infty$$ where W_∞ is work that force does to bring in from ∞.

QUESTION:



in going from i) to f), does E-field do + or − work? potential energy increase or decrease?

Electric Potential

potential energy depends on the charge magnitude. We would like to remove this dependence, so divide by charge

(similar to how we did to get E-field from force)

For example, suppose we place a test charge of $\frac{1}{2}$ C at a point in E-field where potential energy is 2 J

then $\frac{P.E.}{charge} = \frac{2\ J}{1/2\ C} = 4\ J/C$

Now if we place charge of 2 C at same point, P.E. would be 8 J

but $\frac{P.E.}{charge} = \frac{8\ J}{2\ C} = 4\ J/C$

So $\frac{P.E.}{\text{charge}}$ depends only on electric field & not the charge we place there. Thus, such a quantity is so important that we call it electric potential

$$V = \frac{U}{q} \quad \left(\frac{P.E.}{\text{charge}}\right) \quad (J/C)$$

measured in Volts

potential difference is

$$\Delta V = V_f - V_i = \frac{U_f - U_i}{q} = \frac{\Delta U}{q}$$

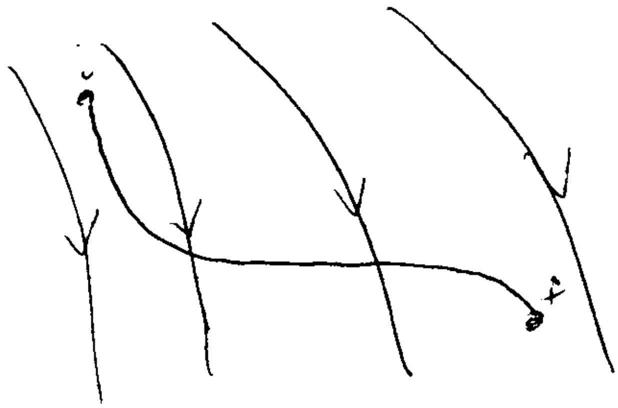
$$= -\frac{W}{q}$$

w/ convention from before we can define

$$V = -\frac{W_\infty}{q}$$

8a

E-field lines



Electric potential difference between two points

$$dW = \vec{F} \cdot d\vec{s}$$

$$\Rightarrow \quad dW = q_0 \vec{E} \cdot d\vec{s}$$

$$\Rightarrow \quad W = \int_i^f dW = \int_i^f q_0 \vec{E} \cdot d\vec{s}$$

$$\Rightarrow \quad \Delta V = V_f - V_i = -\frac{W}{q_0} = -\int_i^f \vec{E} \cdot d\vec{s}$$

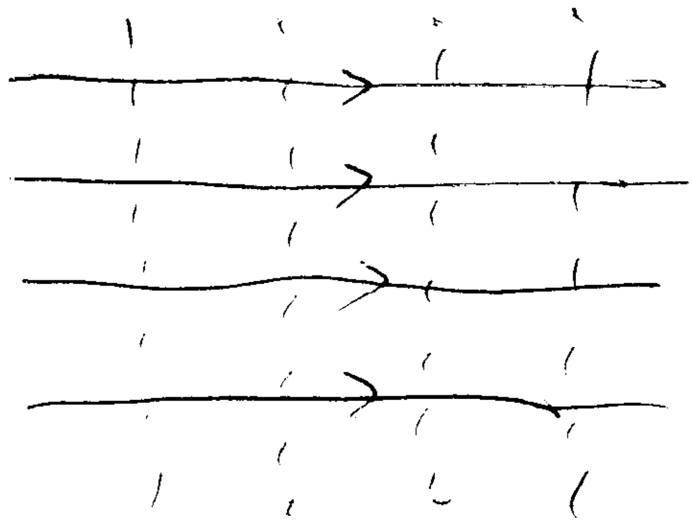
W is work done on a charge by the field

9

electron-Volt is a small unit of energy relevant for atomic scales
- work needed to move an electron through a potential difference of 1V

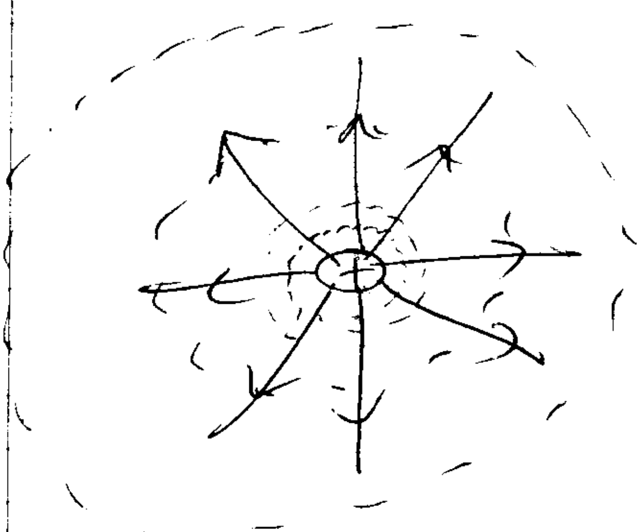
equipotential lines

for uniform field



just as hard to bring in to any point on equipotential line

also, no net work done when moving along an equipotential line.



drops off as $\propto \frac{1}{r}$

equipotential lines are always perpendicular to the direction of field