Lecture 9

15 SEP 2014

Ch. 23 - Gauss' Law

This law is helpful for simplifying calculations of charge distributions that have symmetry (mostly what we deal w/ in this course)

To calculate electric field before, we would consider differential E-field $d\vec{E}$ & integrate. Of course this works, but it can be tedious & there is a method to simplify...

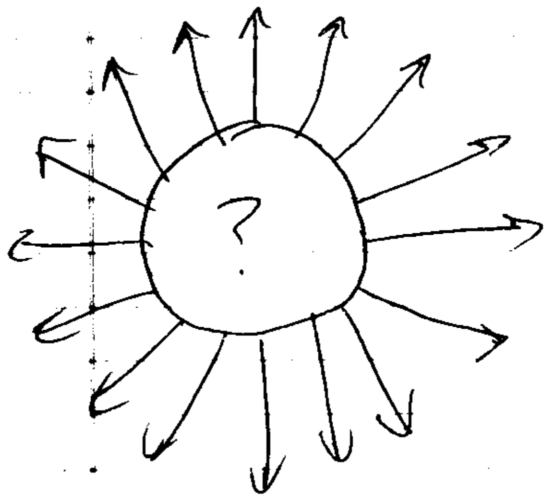
Gauss' law involves using what we call a <u>Gaussian surface</u> enclosing the charge distribution (it is a closed surface)

- This can have any shape you want, but it is best to pick its shape so that it matches the symmetry of the charge distribution

~~Gauss' law relates~~

<u>QUESTION:</u> For a point charge, how should we pick the Gaussian surface? as a sphere

If we choose a Gaussian surface as a spherical shell + And that E-field points radially outward at every point on sphere / what can we conclude about?



has to be spherically symmetric distribution & positive.

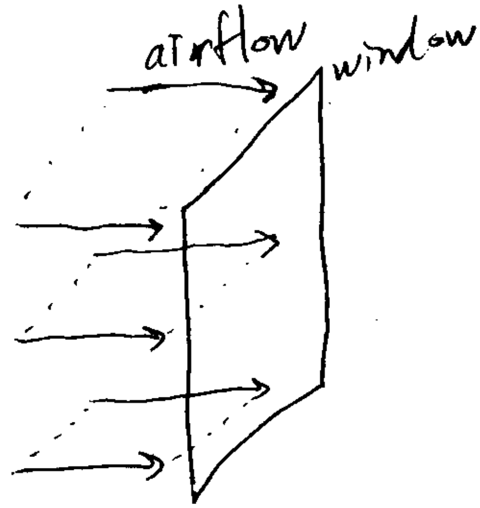
This doesn't tell us how much is inside. For that, we need the ~~notion~~ of electric flux ..

Notion of ~~flux~~ flux

Best analogy is to think of wind flowing through a window.

Let Φ denote volume flow rate (volume per unit time flowing through window)

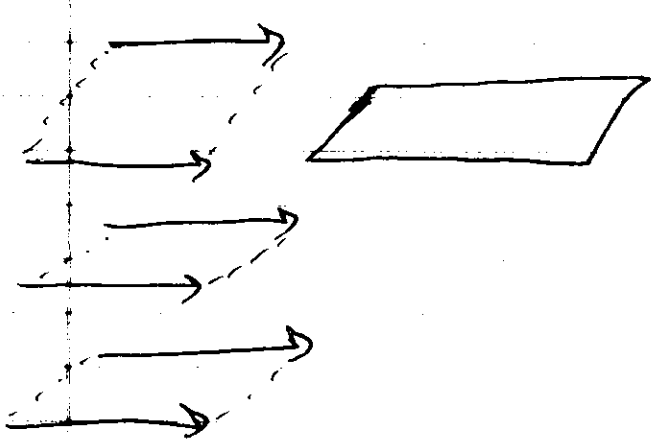
Suppose for simplicity that air flow is uniform



QUESTION: If we know the speed v of air flow + area A of window, what is Φ?

$$\Phi = v \cdot A$$

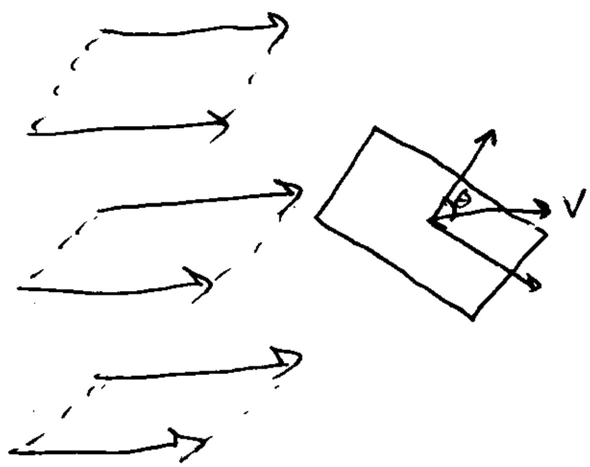
Now, what if we change angle of window



what is Φ?

it is zero.

air skims by but nothing actually goes through window what if window is at some other angle θ wrt air flow?



θ is angle between velocity vector & a vector normal to the window

(normal is orthogonal to all vectors in window)

component of v in same direction as normal of A is

$v\cos\theta$

So this becomes like the 1st scenario

& $\Phi = A v \cos\theta$

can also write this in terms of vectors as

$$\Phi = \vec{v} \cdot \vec{A}$$

where $\vec{A}$ is an area vector that is normal to the ~~area~~ window of interest & whose magnitude is equal to the area of the window

we can also think of air flow as a velocity field &

$\Phi = \vec{V} \cdot \vec{A}$ gives the flux of velocity field through the window

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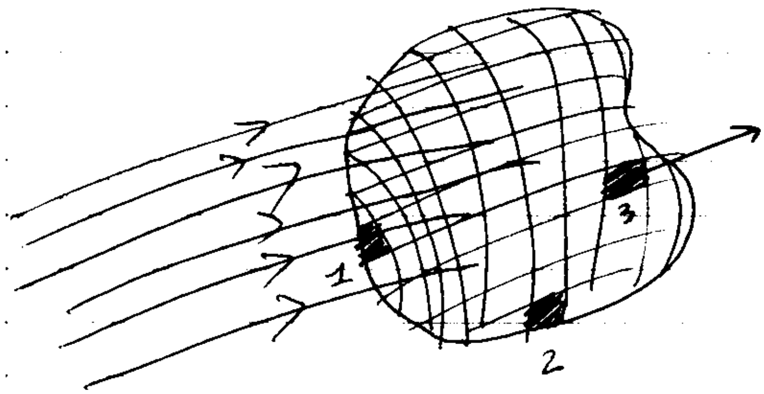
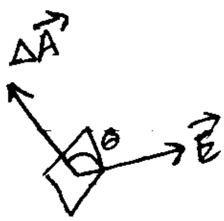
Flux of electric field is very similar

It is taken wrt a Gaussian surface

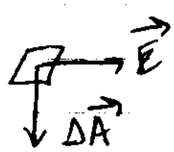
1) Divide the Gaussian surface into very small windows, each w/ their own area vector
2) Find the flux Φ for each window
3) Sum them all up to get the total flux.



1) 

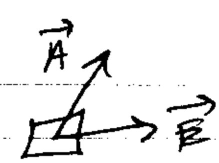
Φ for this window < 0

field is going inward

2) 

field is skimming by

$\Rightarrow$ zero flux

3) 

field going outward

$\Rightarrow$ Φ for this window > 0

to get total flux

$$\Phi = \sum \vec{E} \cdot \Delta\vec{A}$$

get flux for each small window & then sum them all up.

Definition for flux follows by letting windows become infinitesmally small, so that area vectors approach a differential limit $d\vec{A}$ +

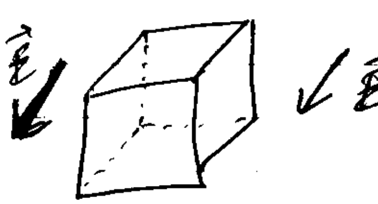
$$\Phi = \oint \vec{E} \cdot d\vec{A}$$

$\oint$ means an integral over a closed surface

$\vec{E}$ is the field at the different locations on the surface

$d\vec{A}$ represents infinitesmally small area vectors all over the surface

QUESTION:



Gaussian cube w/ face area A in a uniform electric field

Given E + A, what is flux through

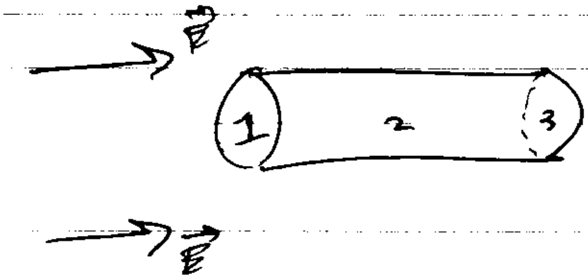
a) the front face EA

b) rear face $-EA$

c) top face 0

d) whole cube 0

QUESTION:



uniform E-field what is flux through cylinder using

$$\Phi = \int \vec{E} \cdot d\vec{A}$$

can break into 3 parts (surfaces)

$$\int_1 \vec{E} \cdot d\vec{A} + \int_2 \vec{E} \cdot d\vec{A} + \int_3 \vec{E} \cdot d\vec{A}$$

$$= -E \cdot A + 0 + E \cdot A$$

(every differential area is antialigned w/ field)

(every differential area orthogonal to field)

(every $d\vec{A}$ aligned w/ field)

Gauss' Law:

net flux Φ of E-field through a closed surface is proportional to net change enclosed by surface

$$\Phi = \frac{q_{enc}}{\varepsilon_0}$$