Coherent communication with continuous quantum variables

Mark M. Wilde,[1,*] Hari Krovi,[2] and Todd A. Brun[2]

[1]*Signal and Image Processing Institute, Department of Electrical Engineering, University of Southern California,*
Los Angeles, California 90089, USA

[2]*Communication Sciences Institute, Department of Electrical Engineering, University of Southern California,*
Los Angeles, California 90089, USA

The coherent bit (cobit) channel is a resource intermediate between classical and quantum communication. It produces coherent versions of teleportation and superdense coding. We extend the cobit channel to continuous variables by providing a definition of the coherent nat (conat) channel. We construct several coherent protocols that use both a position-quadrature and a momentum-quadrature conat channel with finite squeezing. Finally, we show that the quality of squeezing diminishes through successive compositions of coherent teleportation and superdense coding.

The coherent bit (cobit) channel is a useful resource for quantum communication with discrete variables (DV) [1]. The cobit channel Δ_{σ_Z} copies σ_Z eigenstates coherently from Alice to Bob: $|i\rangle^A \rightarrow |i\rangle^A |i\rangle^B$. "Coherence" in this context is synonymous with linearity—the maintenance and linear transformation of superposed states. We name the cobit channel Δ_{σ_Z} the Pauli-Z cobit channel. One can similarly define the Pauli-X cobit channel Δ_{σ_X} that coherently copies σ_X eigenstates.

In this paper, we extend the notion of the cobit channel to continuous-variable (CV) quantum information processing [2,3]. We name the CV version the "conat channel" in analogy with Shannon's name for the information in a continuous random variable measured in units of the natural logarithm. We then construct several coherent protocol primitives. We lastly address duality under resource reversal and discover a difference between the DV and CV coherent channels due to finite squeezing.

What is coherent communication useful for? Insights into quantum protocols occur by replacing classical bits with cobits—replacing a measurement and feedforward classical communication with a coherent channel and replacing a conditional unitary with a controlled unitary. Coherent teleportation and superdense coding for DVs are dual under resource reversal [1,4]. Two protocols are dual under resource reversal if one protocol generates the same resources that the other protocol consumes and vice versa. Coherent remote state preparation (RSP) requires less entanglement than standard RSP [1]. Replacing classical bits with cobits produces coherent versions of several quantum information theory protocols [5,6]. Coherent communication also provides an alternate construction of the newly discovered entanglement-assisted quantum error correcting codes [7,8].

We structure this paper by first motivating and providing a general Heisenberg-representation definition of a position-quadrature (PQ) conat channel and momentum-quadrature (MQ) conat channel. We then construct examples of CV coherent protocols. Finally, we analyze the duality of coherent teleportation and coherent superdense coding under resource

reversal. We find that finitely squeezed coherent CV teleportation and superdense coding are dual under resource reversal only for some maximum number of compositions; beyond that point, classical operations suffice to implement the effective protocol. Duality does not hold when the number of compositions exceeds the maximum.

The cobit-channel definition immediately tempts one to define an ideal PQ conat channel as the quantum-feedback operation Δ_X which copies position eigenstates $|x\rangle^A \rightarrow |x\rangle^A |x\rangle^B$. The ideal MQ conat channel is the operation Δ_P that copies momentum eigenstates. We call these conat channels "ideal" because copying the eigenstates exactly requires infinite energy.

We provide Heisenberg-representation definitions of both a finitely squeezed PQ conat channel $\widetilde{\Delta}_X$ and MQ conat channel $\widetilde{\Delta}_P$ as an approximation to the above ideal scenarios. The first requirement for $\widetilde{\Delta}_X$ is that it approximate the ideal position-copying behavior mentioned above. $\widetilde{\Delta}_X$ should copy the PQ as exactly as possible given finite squeezing. Observe the effect of the ideal PQ conat channel Δ_X on a momentum eigenstate $|p\rangle$. The resulting two-mode state is a maximally entangled Bell state $\int e^{ipx}|x\rangle^A|x\rangle^B dx$. It is an eigenstate of the total momentum operator $\hat{p}_A + \hat{p}_B$ with eigenvalue p. The second requirement is that the total momentum $\hat{p}_A + \hat{p}_B$ should be close to the original momentum $\hat{p}_A$.

An ϵ-approximate PQ conat channel $\widetilde{\Delta}_X$ performs the following transformation with conditions:

$$[\hat{x}_A \; \hat{p}_A]^T \overset{\widetilde{\Delta}_X}{\rightarrow} [\hat{x}_{A'} \; \hat{p}_{A'} \; \hat{x}_{B'} \; \hat{p}_{B'}]^T \tag{1}$$

$$[\hat{x}_{A'}, \hat{p}_{A'}] = [\hat{x}_{B'}, \hat{p}_{B'}] = i, \tag{}$$

$$\hat{x}_{A'} = \hat{x}_A, \tag{2}$$

$$\hat{x}_{B'} = \hat{x}_A + \hat{x}_{\Delta_X}, \tag{3}$$

$$\hat{p}_{A'} = \hat{p}_A + \hat{p}_{\Delta_X}, \tag{4}$$

$$\langle \hat{x}_{\Delta_X} \rangle = \langle \hat{p}_{\Delta_X} + \hat{p}_{B'} \rangle = 0, \tag{5}$$

*mark.wilde@usc.edu

$$\langle \hat{x}_{\Delta_X}^2 \rangle, \quad \langle (\hat{p}_{\Delta_X} + \hat{p}_{B'})^2 \rangle \lesssim \epsilon. \tag{6}$$

The momentum quadrature $\hat{p}_{B'}$ is arbitrary as long as it obeys the above constraints. An ϵ-approximate MQ conat channel $\tilde{\Delta}_P$ performs the following transformation with conditions:

$$[\hat{x}_A \ \hat{p}_A]^T \xrightarrow{\tilde{\Delta}_P} [\hat{x}_{A''} \ \hat{p}_{A''} \ \hat{x}_{B''} \ \hat{p}_{B''}]^T$$

$$[\hat{x}_{A''}, \hat{p}_{A''}] = [\hat{x}_{B''}, \hat{p}_{B''}] = i,$$

$$\hat{p}_{A''} = \hat{p}_A,$$

$$\hat{p}_{B''} = \hat{p}_A + \hat{p}_{\Delta_P},$$

$$\hat{x}_{A''} = \hat{x}_A + \hat{x}_{\Delta_P},$$

$$\langle \hat{p}_{\Delta_P} \rangle = \langle \hat{x}_{\Delta_P} + \hat{x}_{B''} \rangle = 0,$$

$$\langle \hat{p}_{\Delta_P}^2 \rangle, \quad \langle (\hat{x}_{\Delta_P} + \hat{x}_{B''})^2 \rangle \lesssim \epsilon. \tag{7}$$

The position quadrature $\hat{x}_{B''}$ is arbitrary as long as it obeys the above constraints. We require $0 < \epsilon < 1$ for both conat channels.

Fourier transformation gives the relationship between a MQ and PQ conat channel: $\tilde{\Delta}_P = (\mathbb{F}^{-1} \otimes \mathbb{F}^{-1}) \tilde{\Delta}_X \mathbb{F}$. Both a PQ and MQ conat channel implement coherent teleportation— just as Braunstein and Kimble use both PQ and MQ homodyne detection in their teleportation scheme [9]. Our coherent teleportation protocol for CVs is similar to theirs except that a PQ and MQ conat channel replaces the feedforward classical communication and the PQ and MQ homodyne measurement, respectively.

Coherent teleportation protocols using the above ϵ-approximate conat channels have a coherent-state-averaged fidelity greater than the preparation-and-measurement limit [9] of $1/2$ if $\epsilon < 1$. We consider this limit of $1/2$ as opposed to other limits [10,11] because surpassing this limit implies the use of an entangled resource for teleporting an arbitrary coherent state. We simply wish to relate the measure of conat-channel performance to the presence of entanglement.

We can measure PQ conat-channel performance by sending its two output modes through a 50:50 beamsplitter and determining the second moments of one output's PQ and the other output's MQ. Both outputs—the relative position and total momentum—should have second moment bounded by ϵ and $\langle \hat{p}_A^2 \rangle + \epsilon$, respectively in order to be an ϵ-approximate PQ conat channel.

The above definitions are sufficient for implementing a coherent teleportation with CVs via conat channels. They are also necessary for realizing two conat channels as a result of a coherent superdense coding.

We define a two-mode system with Heisenberg-picture quadrature operators $\hat{x}_A$, $\hat{p}_A$, $\hat{x}_B$, $\hat{p}_B$ as ϵ-position-correlated if $\langle (\hat{x}_A - \hat{x}_B)^2 \rangle \lesssim \epsilon$ and $\langle (\hat{p}_A + \hat{p}_B)^2 \rangle \lesssim \epsilon$. It is ϵ-momentum-correlated if $\langle (\hat{x}_A + \hat{x}_B)^2 \rangle \lesssim \epsilon$ and $\langle (\hat{p}_A - \hat{p}_B)^2 \rangle$
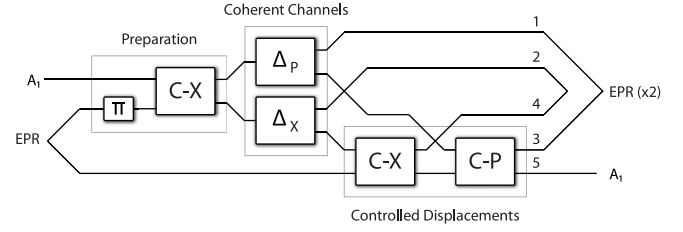

FIG. 1. Coherent teleportation. π is a reflection operation. C-X and C-P are a controlled-position displacement and a controlled-momentum displacement respectively.

$\lesssim \epsilon$. It is ϵ-position-entangled or ϵ-momentum-entangled if $\epsilon < 1$ [12].

We use several operations throughout this Letter. A reflection reverses the quadrature operators of a single mode: $\hat{x} \rightarrow -\hat{x}$, $\hat{p} \rightarrow -\hat{p}$. A controlled-position displacement is a two-mode operation: $\hat{x}_1 \rightarrow \hat{x}_1$, $\hat{p}_1 \rightarrow \hat{p}_1 - \hat{p}_2$, $\hat{x}_2 \rightarrow \hat{x}_2 + \hat{x}_1$, $\hat{p}_2 \rightarrow \hat{p}_2$. A controlled-momentum displacement is as follows: $\hat{x}_1 \rightarrow \hat{x}_1 - \hat{x}_2$, $\hat{p}_1 \rightarrow \hat{p}_1$, $\hat{x}_2 \rightarrow \hat{x}_2$, $\hat{p}_2 \rightarrow \hat{p}_2 + \hat{p}_1$.

Vaidman provided the first theoretical description of CV teleportation [13] followed by Braunstein and Kimble's in terms of the quadratures of the EM field [9]. We construct a coherent version of Braunstein and Kimble's protocol: coherent teleportation (Fig. 1). The protocol is similar to Harrow's for DVs [1]. Suppose Alice wants to teleport a quantum state A_1 to Bob coherently via PQ and MQ conat channels. They possess two modes A and B that are ϵ_1-position-correlated with the additional restriction that they have mean-zero relative position and total momentum $\langle \hat{x}_A - \hat{x}_B \rangle = \langle \hat{p}_A + \hat{p}_B \rangle = 0$. Alice performs a reflection on her mode A followed by a controlled-position displacement on her two modes A_1 and A. These two operations replace the beamsplitter in Braunstein and Kimble's teleportation protocol. She sends her first mode through an ϵ_2-approximate MQ conat channel $\tilde{\Delta}_P$ and her second mode through an ϵ_3-approximate PQ conat channel $\tilde{\Delta}_X$. The two conat channels $\tilde{\Delta}_X$ and $\tilde{\Delta}_P$ replace the feedforward classical communication and position-quadrature and momentum-quadrature homodyne measurements, respectively. The global state becomes a five-mode state. Alice possesses her two original modes and Bob possesses two additional modes due to both conat channels. Bob performs a controlled-position and controlled-momentum displacement according to Fig. 1. These controlled displacements replace the conditional displacements in the original protocol. The five modes then have the Heisenberg-picture observables

$$\hat{x}_1 = \hat{x}_{A_1} + \hat{x}_{\Delta_P}, \quad \hat{p}_1 = \hat{p}_A + \hat{p}_{A_1},$$

$$\hat{x}_2 = \hat{x}_{A_1} - \hat{x}_A, \quad \hat{p}_2 = -\hat{p}_A + \hat{p}_{\Delta_X},$$

$$\hat{x}_3 = (\hat{x}_A - \hat{x}_B) - \hat{x}_{A_1} + \hat{x}_{B''} - \hat{x}_{\Delta_X},$$

$$\hat{p}_3 = \hat{p}_A + \hat{p}_{A_1} + \hat{p}_{\Delta_P},$$

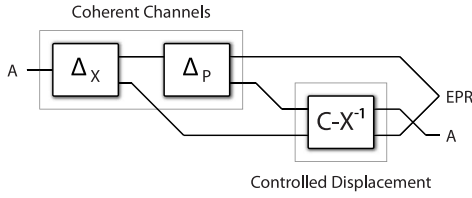$$\hat{x}_4 = \hat{x}_{A_1} - \hat{x}_A + \hat{x}_{\Delta_X}, \quad \hat{p}_4 = \hat{p}_{B'} - \hat{p}_B,$$

FIG. 2. Alternate coherent teleportation protocol.

$$\hat{x}_5 = \hat{x}_{A_1} + (\hat{x}_B - \hat{x}_A) + \hat{x}_{\Delta_X},$$

$$\hat{p}_5 = \hat{p}_{A_1} + (\hat{p}_A + \hat{p}_B) + \hat{p}_{\Delta_P}. \tag{8}$$

Bob possesses the teleported state—Alice's original mode A_1—in mode five. The coherent-state-averaged teleportation fidelity F [14] is as follows:

$$F = 2/[(\langle(\Delta \hat{x}_{\mathrm{tel}})^2\rangle + 1)(\langle(\Delta \hat{p}_{\mathrm{tel}})^2\rangle + 1)]^{1/2}, \tag{9}$$

where $(\hat{x}_{\mathrm{tel}}, \hat{p}_{\mathrm{tel}})$ is the teleported mode. A lower bound on the fidelity F using the above coherent teleportation protocol is $2/[(2 + \epsilon_1 + \epsilon_2)(2 + \epsilon_1 + \epsilon_3)]^{1/2}$. Suppose the ϵ_1-position-correlated state is entangled so that $\epsilon_1 < 1$. Then the coherent teleportation protocol exceeds the classical limit of $1/2$ [9] if both $\epsilon_2 < 1$ and $\epsilon_3 < 1$. Alice and Bob possess an $(\epsilon_1 + \epsilon_2 + \epsilon_3)$-position-correlated state shared between modes one and three. They also possess an $(\epsilon_1 + \epsilon_3)$-momentum-correlated state shared between modes two and four. Thus the original protocol becomes coherent with the benefit of generating two sets of entanglement correlations if $\epsilon_1 + \epsilon_2 + \epsilon_3 < 1$.

We provide an alternate coherent teleportation protocol which is not a direct mapping to Kimble and Braunstein's scheme (Fig. 2). This protocol is similar to van Enk's C1 protocol [15] and to another protocol [16]. Alice possesses a mode A that she wishes to teleport to Bob using two conjugate conat channels. Alice first sends her mode through an ϵ_1-approximate PQ conat channel. She then sends her mode through an ϵ_2-approximate MQ conat channel. Bob possesses two modes after the two operations. He performs an inverse controlled-position displacement on his two modes. The three modes after the protocol have the Heisenberg-picture observables

$$\hat{x}_1 = \hat{x}_A + \hat{x}_{\Delta_P}, \quad \hat{p}_1 = \hat{p}_A + \hat{p}_{\Delta_X},$$

$$\hat{x}_2 = \hat{x}_A + \hat{x}_{\Delta_X}, \quad \hat{p}_2 = \hat{p}_A + (\hat{p}_{\Delta_X} + \hat{p}_{B'}) + \hat{p}_{\Delta_P},$$

$$\hat{x}_3 = \hat{x}_{B''} - (\hat{x}_A + \hat{x}_{\Delta_X}), \quad \hat{p}_3 = \hat{p}_A + \hat{p}_{\Delta_X} + \hat{p}_{\Delta_P}. \tag{10}$$

Alice possesses the first mode and Bob possesses the last two. Mode two is the teleported mode containing Alice's original state A. A lower bound on the fidelity F is $2/(2 + \epsilon_1 + \epsilon_2)$. The teleportation fidelity exceeds the classical limit of $1/2$ if both $\epsilon_1 < 1$ and $\epsilon_2 < 1$. Alice and Bob possess an $(\epsilon_1 + \epsilon_2)$-momentum-correlated state shared between modes one and three (momentum-entangled if $\epsilon_1 + \epsilon_2 < 1$).

Braunstein and Kimble provided a theoretical proposal for a superdense coding protocol with CVs [17]. They demon-
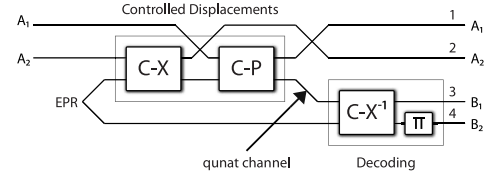

FIG. 3. Coherent superdense coding implements two conat channels.

strated that bipartite CV entanglement and a qunat channel can double classical communication capacity in the limit of large squeezing. We provide a coherent version of their superdense coding protocol by implementing both a PQ and a MQ conat channel rather than two classical nat channels (Fig. 3).

The global state shared between Alice and Bob at the start of the protocol is a four-mode state. Alice possesses two modes A_1 and A_2. She wants to simulate a MQ conat channel on A_1 and a PQ conat channel on A_2. Alice and Bob possess an ϵ-position-correlated state shared between modes three $(\hat{x}_A, \hat{p}_A)$ and four $(\hat{x}_B, \hat{p}_B)$ with mean-zero relative position and total momentum. Alice performs a controlled-position displacement on her modes two and three followed by a controlled-momentum displacement on modes one and three. The controlled displacements replace the conditional displacements in Braunstein and Kimble's protocol. Alice sends mode three to Bob via the qunat channel. Bob performs an inverse controlled-position displacement on modes three and four followed by reflecting mode four. The last two operations replace the beamsplitter in the original dense coding protocol. The last two operations are also the inverse operations of the preparation stage for the coherent teleportation protocol (compare Fig. 1 and Fig. 3). The four modes then have the Heisenberg-picture observables

$$\hat{x}_1 = \hat{x}_{A_1} - (\hat{x}_{A_2} + \hat{x}_A), \quad \hat{p}_1 = \hat{p}_{A_1},$$

$$\hat{x}_2 = \hat{x}_{A_2}, \quad \hat{p}_2 = \hat{p}_{A_2} - \hat{p}_A,$$

$$\hat{x}_3 = \hat{x}_{A_2} + \hat{x}_A, \quad \hat{p}_3 = \hat{p}_{A_1} + (\hat{p}_A + \hat{p}_B),$$

$$\hat{x}_4 = \hat{x}_{A_2} + (\hat{x}_A - \hat{x}_B), \quad \hat{p}_4 = -\hat{p}_B. \tag{11}$$

Modes one and three satisfy the conditions for an ϵ-approximate MQ conat channel. Modes two and four satisfy the conditions for an ϵ-approximate PQ conat channel. Coherent superdense coding thus gives an operational interpretation to the PQ and MQ conat channels.

Coherent teleportation and superdense coding for DVs are dual under resource reversal [1,4], meaning that the resources generated by one protocol are consumed by the other and vice versa. Duality under resource reversal is only possible to some degree with CVs because of finite squeezing. We first give two ways of composing the protocols. We then illustrate how the duality does not hold for some finite number of compositions.

Implementing coherent teleportation with coherent superdense coding is one way of composition. Coherent super-

dense coding (Fig. 3) plays the role of the coherent channels in coherent teleportation (Fig. 1). Alice wishes to teleport a mode A_1 to Bob. Suppose Alice and Bob share an ϵ_1-position-correlated state with mode operators $(\hat{x}_A, \hat{p}_A, \hat{x}_B, \hat{p}_B)$ and an ϵ_2-position-correlated state with mode operators $(\hat{x}_A^-, \hat{p}_A^-, \hat{x}_B^-, \hat{p}_B^-)$. Both correlated sets have mean-zero relative position and total momentum. They use the first correlated state for coherent teleportation. They use the second correlated state for coherent superdense coding—which in turn implements both an ϵ_2-approximate MQ conat channel and an ϵ_2-approximate PQ conat channel. The results are the same as coherent teleportation (Fig. 1) with some modifications. Alice and Bob possess an $(\epsilon_1 + \epsilon_2)$-momentum-correlated state shared between modes one and three and an $(\epsilon_1 + \epsilon_2)$-position-correlated state shared between modes two and four. Mode five is the teleported mode with Alice's original state A_1. A lower bound on the fidelity F using this protocol is $2/(2 + \epsilon_1 + \epsilon_2)$. The fidelity exceeds the limit of $1/2$ if both $\epsilon_1 < 1$ and $\epsilon_2 < 1$.

Implementing coherent superdense coding with coherent teleportation is another way of composition. The alternate coherent teleportation in Fig. 2 replaces the qunat channel in Fig. 3. The protocol is similar to coherent superdense coding except for an additonal set of two correlated modes. It begins with Alice possessing two modes represented by quadrature operators $(\hat{x}_{A_1}, \hat{p}_{A_1}, \hat{x}_{A_2}, \hat{p}_{A_2})$. She wishes to implement a PQ and MQ conat channel on these two modes. Suppose that Alice and Bob possess an ϵ_1-position-correlated state represented by the quadrature operators $(\hat{x}_A, \hat{p}_A, \hat{x}_B, \hat{p}_B)$ with mean-zero relative position and total momentum. Alice performs controlled displacements on her three modes (Fig. 3). She uses alternate coherent teleportation to implement the qunat channel. She uses an ϵ_2-approximate PQ conat channel and an ϵ_3-approximate MQ conat channel for alternate coherent teleportation. Bob performs the last two operations in Fig. 3. The six modes then have the Heisenberg-picture observables

$$\hat{x}_1 = \hat{x}_{A_1} - (\hat{x}_{A_2} + \hat{x}_A), \quad \hat{p}_1 = \hat{p}_{A_1},$$

$$\hat{x}_2 = \hat{x}_{A_2}, \quad \hat{p}_2 = \hat{p}_{A_2} - \hat{p}_A,$$

$$\hat{x}_3 = \hat{x}_{A_2} + \hat{x}_A + \hat{x}_{\Delta_P}, \quad \hat{p}_3 = \hat{p}_{A_1} + \hat{p}_A + \hat{p}_{\Delta_X},$$

$$\hat{x}_4 = \hat{x}_{A_2} + \hat{x}_A + \hat{x}_{\Delta_X},$$

$$\hat{p}_4 = \hat{p}_{A_1} + (\hat{p}_A + \hat{p}_B) + (\hat{p}_{\Delta_X} + \hat{p}_{B'}) + \hat{p}_{\Delta_P},$$

$$\hat{x}_5 = \hat{x}_{B''} - (\hat{x}_{A_2} + \hat{x}_A + \hat{x}_{\Delta_X}),$$

$$\hat{p}_5 = \hat{p}_{A_1} + \hat{p}_A + \hat{p}_{\Delta_X} + \hat{p}_{\Delta_P},$$

$$\hat{x}_6 = \hat{x}_{A_2} + (\hat{x}_A - \hat{x}_B) + \hat{x}_{\Delta_{X'}}, \quad \hat{p}_6 = -\hat{p}_B, \quad (12)$$

where Alice possesses modes 1–3 and Bob possesses modes 4–6. Alice and Bob share an $(\epsilon_2 + \epsilon_3)$-momentum-correlated state between modes three and five. Alice implements an $(\epsilon_1 + \epsilon_2 + \epsilon_3)$-approximate MQ conat channel between modes one and four if $\epsilon_1 + \epsilon_2 + \epsilon_3 < 1$. She implements an $(\epsilon_1 + \epsilon_2)$-approximate PQ conat channel between modes two and six if $\epsilon_1 + \epsilon_2 < 1$.

Coherent teleportation and coherent superdense coding are dual under resource reversal in the sense given by the above first-order compositions if the ϵ quantities are small enough. But examine the above protocols to observe a loss of duality. The ϵ quantities accumulate additively when composing multiple protocols using imperfect conat channels. Repeated use of nonideal conat channels eventually degrades the available level of squeezing until the sufficient conditions for entanglement and for teleportation fidelity exceeding the classical limit no longer hold. The protocols are not dual under resource reversal after some maximum number of compositions due to finite-squeezing losses.

We provided conat-channel definitions and demonstrated several coherent protocols. We concluded with an analysis of the duality under resource reversal of coherent teleportation and coherent superdense coding. The conat channel should lead to other CV coherent protocols.

[1] A. Harrow, Phys. Rev. Lett. **92**, 097902 (2004).

[2] *Quantum Information with Continuous Variables*, edited by S. L. Braunstein and A. Pati (Springer, 2003).

[3] S. L. Braunstein and P. van Loock, Rev. Mod. Phys. **77**, 513 (2005).

[4] I. Devetak, Phys. Rev. Lett. **97**, 140503 (2006).

[5] I. Devetak, A. W. Harrow, and A. Winter, Phys. Rev. Lett. **93**, 230504 (2004).

[6] I. Devetak, A. W. Harrow, and A. Winter, e-print arXiv:quant-ph/0512015.

[7] T. A. Brun, I. Devetak, and M.-H. Hsieh, Science **314**, 436 (2006).

[8] T. A. Brun, I. Devetak, and M.-H. Hsieh, e-print arXiv:quant-ph/0608027.

[9] S. L. Braunstein and H. J. Kimble, Phys. Rev. Lett. **80**, 869 (1998).

[10] C. M. Caves and K. Wodkiewicz, Phys. Rev. Lett. **93**, 040506 (2004).

[11] F. Grosshans and P. Grangier, Phys. Rev. A **64**, 010301(R) (2001).

[12] L.-M. Duan, G. Giedke, J. I. Cirac, and P. Zoller, Phys. Rev. Lett. **84**, 2722 (2000).

[13] L. Vaidman, Phys. Rev. A **49**, 1473 (1994).

[14] A. Furusawa, J. L. Srensen, S. L. Braunstein, C. A. Fuchs, H. J. Kimble, and E. S. Polzik, Science **282**, 706 (1998).

[15] S. J. van Enk, Phys. Rev. A **71**, 032339 (2005).

[16] I. Devetak (private communication).

[17] S. L. Braunstein and H. J. Kimble, Phys. Rev. A **61**, 042302 (2000).