

Addressing the Clumsiness Loophole in a Leggett-Garg Test of Macrorealism

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Abstract The rise of quantum information theory has lent new relevance to experimental tests for non-classicality, particularly in controversial cases such as adiabatic quantum computing superconducting circuits. The Leggett-Garg inequality is a “Bell inequality in time” designed to indicate whether a single quantum system behaves in a macrorealistic fashion. Unfortunately, a violation of the inequality can only show that the system is *either* (i) non-macrorealistic *or* (ii) macrorealistic but subjected to a measurement technique that happens to disturb the system. The “clumsiness” loophole (ii) provides reliable refuge for the stubborn macrorealist, who can invoke it to brand recent experimental and theoretical work on the Leggett-Garg test inconclusive. Here, we present a revised Leggett-Garg protocol that permits one to conclude that a system is *either* (i) non-macrorealistic *or* (ii) macrorealistic but with the property that two seemingly non-invasive measurements can somehow collude and strongly disturb the system. By providing an explicit check of the invasiveness of the measurements, the protocol replaces the clumsiness loophole with a significantly smaller “collusion” loophole.

Keywords Leggett-Garg inequality · Macrorealism · Non-invasive measurability · Clumsiness loophole

1 Introduction

One of the hallmarks of the quantum theory is that it defies our intuition about the world. Faced with experimental manifestations of entanglement and wave-particle

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duality, one may recognize the need to revise classical theories, but one does not readily accept the radical quantum picture of nature. Bell's theorem [1, 2] provided a clear experimental protocol to test whether nature could obey a revised classical theory at least satisfying the minimal postulates of local realism. The experimental demonstration of a violation of Bell's inequality [3] showed that even these minimal postulates must be abandoned.

Unfortunately, the Bell protocol can be experimentally demanding, especially for large quantum systems such as superconducting qubits, requiring two parts that can be entangled, spatially separated, and independently measured. If one wishes to look for signatures of quantum behavior in such systems, a non-local Bell test is usually impractical. Leggett and Garg therefore framed a less experimentally demanding protocol that tests for violation of the postulates of *macrorealism* rather than local realism [4, 5]. Refinements exist in the literature, but the original postulates of macrorealism are as follows: a macroscopic system with two or more macroscopically distinct states available to it will at all times be in one or the other of these states (*macroscopic realism per se*), and it is possible, in principle, to determine the state of the system with arbitrarily small perturbation to its subsequent dynamics (*non-invasive measurability*). Leggett and Garg derived an inequality similar to that of Bell [4] to bound the temporal correlations observable in a macrorealistic theory. (Some have referred to the Leggett-Garg inequality as a *Bell inequality in time*). A violation of the inequality is supposed to show that a system is not behaving macrorealistically.

Despite being more experimentally tractable than a Bell test for systems like superconducting qubits, any Leggett-Garg test suffers from a serious vulnerability in comparison to a Bell test. Since the locality postulate of local realism states that it is impossible to affect a spatially distant physical system, a violation of Bell's inequality can unequivocally demonstrate a failure of local realism. On the other hand, macrorealism does not assert that it is impossible to affect a physical system by measurement but merely that it is possible for a sufficiently adroit measurement to avoid doing so. Thus the Leggett-Garg test can show only that the system is *either* (i) non-macrorealistic *or* (ii) macrorealistic but subjected to a measurement technique that happens to disturb the system. Rather than abandoning a cherished view of the nature of physical reality, an adherent of macrorealism will likely attribute a violation of the Leggett-Garg inequality to the "clumsiness" loophole (ii), which results from experimental limitations, rather than the radical finding (i), which topples the macrorealist's picture of nature. Recent experimental Leggett-Garg tests [6–8] and theoretical work [9–13] are thus inconclusive in the eyes of a stubborn macrorealist. Leggett and Garg acknowledge this loophole but maintain that clever measurement schemes, such as ideal negative-result measurements, argue against the macrorealist's retort.

In this paper, we provide a more methodical and general means of addressing the clumsiness loophole. Of course, it is impossible in principle to prove once and for all that a measurement device is non-invasive. Even if the device were to pass a number of tests for non-invasiveness, one never knows whether some test exists which the device would fail (i.e., a scientific hypothesis like "the measurement device is non-invasive" can be falsified but cannot be proven true once and for all). Furthermore, in our context, we would need to demonstrate that the device would be non-invasive if not for the fact that the quantum system being measured cannot be measured non-invasively. Our approach is instead to frame the notion of an "adroit measurement"

and show how to demonstrate it experimentally. We then devise an experiment that shows how to use only adroit measurements to violate the Leggett-Garg inequality. The result is a compelling protocol that addresses the clumsiness loophole, while having more modest experimental requirements (for systems like superconducting qubits) than the Bell inequality. It allows one to conclude that a system under investigation is *either* (i) non-macrorealistic *or* (ii) macrorealistic but with the property that two adroit measurements can somehow collude and strongly disturb the system. The protocol advances beyond the original Leggett-Garg test by providing an explicit way to check whether the measurement technique happens to disturb the system. The clumsiness loophole is thereby closed, although a significantly smaller collusion loophole remains since the check is not exhaustive.

We structure this paper as follows. We first review the standard Leggett-Garg inequality and then present an ideal variation of it that replaces the clumsiness loophole with the smaller collusion loophole as described above. We show how increasing the number of adroit measurements leads to a stronger violation of the Leggett-Garg inequality. We then generalize the scenario and report how the inclusion of noise leads to a practical trade-off between the number of adroit measurements and the strength of dephasing noise. Our conclusion includes remarks on the clumsiness loophole as it relates to recent weak-measurement versions of the Leggett-Garg inequality [11–13].

2 Leggett-Garg Inequality for Qubits

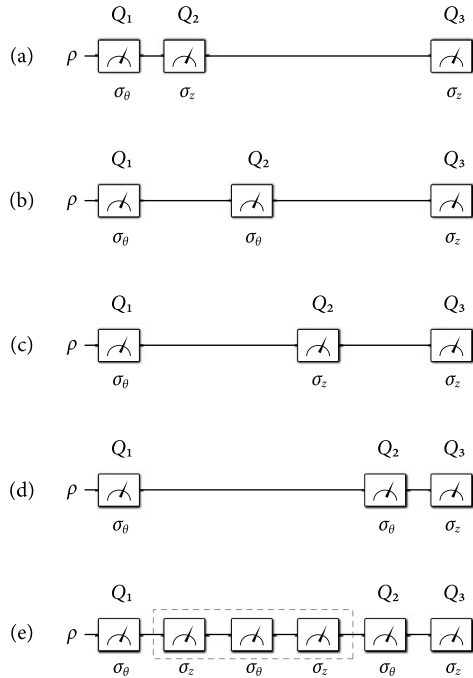
Suppose that we prepare a given system in some specific initial state and measure a dichotomic observable Q (with realizations ± 1) after delays t_i , where $i = 1, 2, 3$. We repeat and average the results to compute the two-time correlation functions $C_{i,j} \equiv \langle Q_i Q_j \rangle$, where we employ the shorthand $Q_i \equiv Q(t_i)$. The Leggett-Garg [4] inequality states

$$\mathcal{L} \equiv C_{1,2} + C_{2,3} + C_{1,3} + 1 \geq 0, \quad (1)$$

for any choice of t_1 , t_2 , and t_3 . As a matter of elementary mathematics, this inequality is always satisfied if we measure three ± 1 -valued numbers Q_1 then Q_2 then Q_3 , compute $Q_1 Q_2$, $Q_2 Q_3$, and $Q_1 Q_3 = (Q_1 Q_2)(Q_2 Q_3)$, repeat and average to obtain $C_{i,j}$. The possibility of a violation only arises if we compute $C_{1,3}$ using a separate series of experiments in which Q_1 and Q_3 are measured but not Q_2 ; one can emphasize this by attaching a ' superscript to $C_{1,3} = \langle Q_1 Q_3 \rangle'$. A violation of (1) can only occur if $\langle Q_1 Q_3 \rangle' \neq \langle Q_1 Q_3 \rangle = \langle (Q_1 Q_2)(Q_2 Q_3) \rangle$. This should only happen if the act of measuring Q_2 “matters”, which should not be the case for a macrorealistic system being measured non-invasively.

Suppose, then, that an experimentalist finds a violation of the inequality. How should this result be interpreted? Perhaps the system is not macrorealistic—a violation of the above inequality occurs, for example, for a two-state quantum system initialized to a maximally mixed state in a noiseless environment with vanishing system Hamiltonian, and $Q_1 = \sigma_z$, $Q_2 = (\sigma_y - \sigma_z)/\sqrt{2}$, $Q_3 = -\sigma_y$. (Here, σ_i denotes

Fig. 1 A series of experiments to address the clumsiness loophole in a violation of the Leggett-Garg inequality in (1). Suppose that the second measurement in (a)–(d) is shown experimentally to have no effect on the joint probability distribution between measurements of observables Q_1 and Q_3 . Then these measurements are adroit, and any sequence of them is also adroit. We then perform a new experiment (e) that includes these measurements. Given the evidence of adroitness provided by (a)–(d), plus the closure of adroitness axiom, violation of the Leggett-Garg inequality in (e) can be attributed to a failure of macrorealism rather than to experimental clumsiness



a Pauli matrix.¹ For such a quantum system, the measurement of Q_2 “matters” because it causes a collapse according to the axioms of quantum mechanics. On the other hand, a violation could also occur for a macrorealistic two-state system being measured in an invasive fashion because of limitations in the experimental measurement technique; in this case the act of measuring Q_2 matters only because experimental practicalities have led to a “clumsy” measurement apparatus. Here, we propose a protocol that is designed to mitigate this stark uncertainty in interpretation.

Consider the experiment depicted in Fig. 1(a). We will say that the second measurement is adroit if it does not have any effect on the joint probability distribution of the outcomes of the first and third measurements. That is, suppose that results a and c correspond to the respective outcomes of the first and third measurements. Then, the second measurement is ϵ -adroit if

$$\sum_{a,c} |P\{a, c \mid \text{2nd measurement is performed}\} - P\{a, c \mid \text{2nd measurement not performed}\}| \leq \epsilon,$$

where we stipulate (as in Fig. 1(a)) that no other measurement be performed besides the first, second, and third measurements. An experimentalist can confirm whether

¹We are assuming a vanishing Hamiltonian for now in order to simplify the presentation. Section 4 details an equivalent, but experimentally more natural case, in which there is a non-trivial Hamiltonian $\omega\sigma_X$, the observables are all $Q(t_i) = \sigma_Z$, and the measurement times t_i are chosen appropriately.

a measurement is ϵ -adroit (we will simply say “adroit” in what follows) by collecting statistics to build confidence that the above condition holds. Assuming that the ϵ -adroitness test gives convincing evidence that the 2nd measurement has no effect on the system, it is natural to assume for a macrorealistic system that performing two ϵ -adroit measurements should yield a 2ϵ -adroit composite measurement (this could be termed the *closure of adroit measurements*). Of course, if the 2nd measurement has some violent effect on the system that is not revealed in the ϵ -adroitness test, it is possible in principle for two seemingly innocuous ϵ -adroit measurements to collude and have a dramatic effect on the system, violating the closure of adroit measurements. Our protocol does not rule out this (in our opinion unnatural) possibility.

With this notion of adroit measurements in mind, consider the procedure depicted in Fig. 1. Suppose the experimentalist first performs experiments (a)–(d) and can demonstrate in each case that the second measurement is adroit as defined above. Finally, the experimentalist performs Fig. 1(e). Experiment (e) follows the Leggett-Garg protocol except that $\langle Q_1 Q_2 \rangle$ and $\langle Q_2 Q_3 \rangle$ are obtained in the presence of the boxed measurements, whereas $\langle Q_1 Q_3 \rangle'$ is obtained not only without performing Q_2 , but also without performing the boxed measurements. Since the boxed measurements and Q_2 of Fig. 1(e) have individually been shown to be adroit in experiments (a)–(d), the closure of adroit measurements implies that it should have negligible effect on the correlator $\langle Q_1 Q_3 \rangle'$ whether they are performed or not.

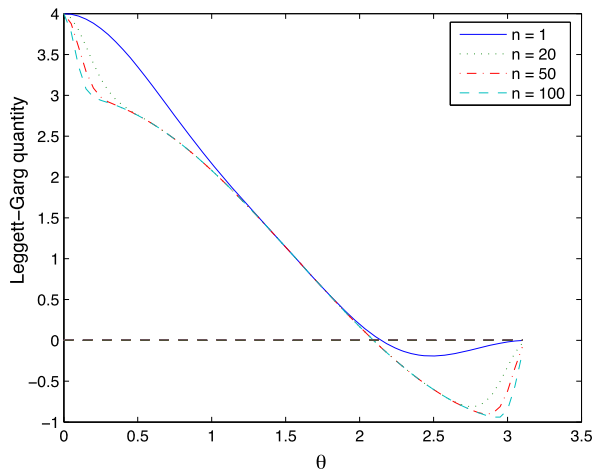
What happens if we apply this new procedure to a two-state system such as a qubit that is actually quantum mechanical? As shown in Fig. 1, we choose as observables σ_z and $\sigma_\theta \equiv \cos(\theta)\sigma_z + \sin(\theta)\sigma_x$. Assume for now that the system is noiseless and has vanishing Hamiltonian. Then, it is clear that the second measurement in experiments (a)–(d) does not affect the correlations between the first and third measurements because measuring a variable twice in a row has no additional effect on the system. Thus, the measurements are perfectly adroit in this ideal case. What happens in experiment (e)? To answer this question, we first calculate several relevant quantities. We define superoperator $\bar{\Delta}$ as a σ_z basis dephasing of a qubit with density operator ρ : $\bar{\Delta}(\rho) \equiv \frac{1}{2}(\rho + \sigma_z \rho \sigma_z)$ and $\bar{\Delta}_\theta$ as a σ_θ basis dephasing: $\bar{\Delta}_\theta(\rho) \equiv \frac{1}{2}(\rho + \sigma_\theta \rho \sigma_\theta)$. The following relation is useful: $\bar{\Delta}(\sigma_\theta) = \cos(\theta)\sigma_z$, and a similar relation can be derived by exploiting it: $\bar{\Delta}_\theta(\sigma_z) = \cos(\theta)\sigma_\theta$. Now, we calculate the correlation $\langle Q_1 Q_3 \rangle' = \frac{1}{2}\text{Tr}[\sigma_z \{\sigma_\theta, \rho\}]$, where $\{\sigma_\theta, \rho\}$ is the anticommutator. The other correlation functions $\langle Q_1 Q_2 \rangle$ and $\langle Q_2 Q_3 \rangle$ are as follows:

$$\langle Q_1 Q_2 \rangle = \frac{1}{2}\text{Tr}[\sigma_\theta (\bar{\Delta} \circ \bar{\Delta}_\theta \circ \bar{\Delta}) (\{\sigma_\theta, \rho\})], \tag{2}$$

$$\langle Q_2 Q_3 \rangle = \frac{1}{2}\text{Tr}[\sigma_z \{ \sigma_\theta, (\bar{\Delta} \circ \bar{\Delta}_\theta)^2 (\rho) \}]. \tag{3}$$

Let us suppose that the qubit begins in the maximally mixed state, so that $\rho = I/2$. Then, the above expressions imply $\langle Q_1 Q_3 \rangle' = \cos(\theta)$, $\langle Q_1 Q_2 \rangle = \cos^4(\theta)$, $\langle Q_2 Q_3 \rangle = \cos(\theta)$. The Leggett-Garg inequality $\mathcal{L} = 1 + \cos^4(\theta) + 2 \cos(\theta) \geq 0$ is violated if we choose θ between $.683\pi$ and π . If one could demonstrate this violation experimentally, it would show that our two-level quantum system is *either* (i) non-macrorealistic *or* (ii) macrorealistic but with the peculiar property that two adroit measurements can somehow collude and strongly disturb the system.

Fig. 2 The above figure depicts the Leggett-Garg quantity in (4) for different values of the number n of interleaved pairs of measurements. The *dashed line* divides the space into two regions: *points above the line* do not violate the Leggett-Garg inequality, while *points below* violate it



3 Generalization to More Measurements

We can generalize the above analysis to the scenario where the experimentalist performs $2n + 1$ measurements in the dotted box. We have σ_θ as Q_1 ; then n interleaved pairs of measurements of σ_z and σ_θ followed by σ_z ; then σ_θ as Q_2 ; and finally σ_z as Q_3 . In this case,

$$\langle Q_1 Q_2 \rangle = \frac{1}{2} \text{Tr} \left[\sigma_z \left(\bar{\Delta} \circ (\bar{\Delta}_\theta \circ \bar{\Delta})^n \right) (\{\sigma_\theta, \rho\}) \right],$$

$$\langle Q_2 Q_3 \rangle = \frac{1}{2} \text{Tr} \left[\sigma_z \left\{ \sigma_\theta, (\bar{\Delta} \circ \bar{\Delta}_\theta)^{2n} (\rho) \right\} \right],$$

while the other correlation function $\langle Q_1 Q_3 \rangle'$ remains the same. The measurements are again perfectly adroit in this ideal case. If ρ is maximally mixed, $\langle Q_1 Q_3 \rangle' = \cos(\theta)$, $\langle Q_1 Q_2 \rangle = \cos^{2(n+1)}(\theta)$, $\langle Q_2 Q_3 \rangle = \cos(\theta)$, then the Leggett-Garg inequality reads as follows:

$$\mathcal{L} = 1 + \cos^{2(n+1)}(\theta) + 2 \cos(\theta) \geq 0. \tag{4}$$

We can take the number n of interleaved pairs of measurements to be arbitrarily large. As shown in Fig. 2, we obtain a somewhat larger violation for a slightly larger range as n increases ($2\pi/3 \leq \theta < \pi$ when $n \rightarrow \infty$). Note that it is not possible to violate the inequality if one *reduces* the number of measurements in the dotted box in (e) from 3 to 1; although our protocol clearly depends upon the non-commutativity of σ_z and σ_θ , one does not obtain an inequality violation from the most straightforward version of our protocol, where the measurements are σ_θ , then σ_z , then σ_θ , then σ_z .

4 Leggett-Garg Inequality with Non-zero Hamiltonian

The original Leggett-Garg inequality was framed for an rf-SQUID system with non-trivial dynamics. A violation of the inequality (1) occurs, for example, for a two-state

quantum system such as an rf-SQUID initialized to a maximally mixed state with system Hamiltonian $\omega\sigma_x/2$, observable Q chosen to be σ_z , and measurement times $t_1 = 0$, $t_2 = 3\pi/4\omega$, and $t_3 = 3\pi/2\omega$. It is straightforward to adapt our protocol to this rf-SQUID case simply by converting Fig. 1 from the Schrödinger picture to the Heisenberg picture. For simplicity, we take Hamiltonian $\omega\sigma_x$ so that the evolution operator is $U_t \equiv \exp\{-i\omega\sigma_x t\}$, where we implicitly set $\hbar = 1$. The following relations hold

$$\begin{aligned} \mathcal{U}_t(\sigma_z) &= \sigma_y \sin(2\omega t) + \sigma_z \cos(2\omega t), \\ \mathcal{U}_t(\sigma_\theta) &= \sigma_x \sin(\theta) + \cos(\theta)(\sigma_y \sin(2\omega t) + \sigma_z \cos(2\omega t)), \end{aligned}$$

where $\mathcal{U}_t(\rho) \equiv U_t \rho U_t^\dagger$. Notice that the second measurements in experiments (a)–(d) are all perfectly adroit if we measure at time intervals equal to $\pi m/\omega$ where m is some positive integer. This adroitness holds because the function $\sin(2\omega t)$ vanishes at these times, and the scenario here then maps to the earlier scenario with trivial dynamics. (In other words, these are quantum nondemolition measurements as in [12].) The correlation functions are as follows:

$$\langle Q_1 Q_3 \rangle' = \frac{1}{2} \text{Tr} [Q_3 \mathcal{U}_{5\tau} (\{Q_1, \mathcal{U}_\tau(\rho)\})], \tag{5}$$

$$\langle Q_1 Q_2 \rangle = \frac{1}{2} \text{Tr} [Q_2 (\mathcal{U}_\tau \circ \bar{\Delta} \circ \mathcal{U}_\tau \circ \bar{\Delta}_\theta \circ \mathcal{U}_\tau \circ \bar{\Delta} \circ \mathcal{U}_\tau) (\{Q_1, \mathcal{U}_\tau(\rho)\})], \tag{6}$$

$$\langle Q_2 Q_3 \rangle = \frac{1}{2} \text{Tr} [Q_3 \mathcal{U}_\tau (\{Q_2, (\mathcal{U}_\tau \circ \bar{\Delta} \circ \mathcal{U}_\tau \circ \bar{\Delta}_\theta \circ \mathcal{U}_\tau \circ \bar{\Delta} \circ \mathcal{U}_\tau \circ \bar{\Delta}_\theta \circ \mathcal{U}_\tau) (\rho)\})], \tag{7}$$

where τ is the uniform time interval between measurements. Choosing ρ as the maximally mixed state, $Q_1 = \sigma_\theta$, $Q_2 = \sigma_\theta$, $Q_3 = \sigma_z$, and each of the time intervals τ equal to $\pi m/\omega$ in (5)–(7) yields the same correlation values as in the trivial dynamics case. Thus, we obtain a violation of the Leggett–Garg inequality if we measure at time intervals τ equal to $\pi m/\omega$ where m is some positive integer and if we choose the angle θ to be in the range given previously.

5 Leggett–Garg Inequality with Dephasing Noise

We modify the above scenarios to include some dephasing effects described by the Lindblad equation [14]:

$$\dot{\rho}(t) = -i[\omega\sigma_x, \rho(t)] + 2\gamma(\sigma_z\rho(t)\sigma_z - \rho(t)), \tag{8}$$

where the Hamiltonian is $\omega\sigma_x$ as before and γ is the rate of dephasing. Let $\mathcal{N}_\tau(\rho)$ denote the time-dependent CPTP map that the above Lindblad equation effects. We can calculate the correlation functions $\langle Q_1 Q_3 \rangle'$, $\langle Q_1 Q_2 \rangle$, and $\langle Q_2 Q_3 \rangle$ by replacing \mathcal{U}_τ with \mathcal{N}_τ in (5)–(7). We also compute the total amount of ϵ -adroitness of the measurements in experiments (a)–(d) of Fig. 1 using the definition of adroitness and

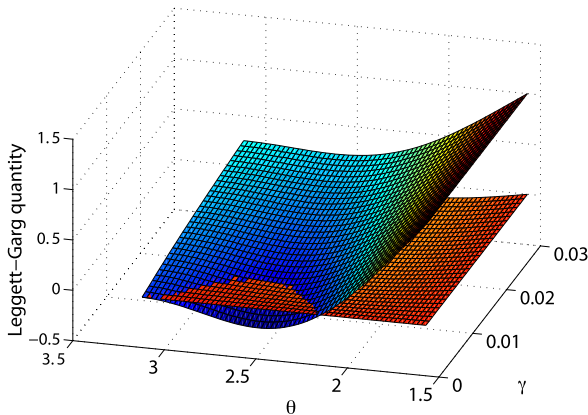


Fig. 3 (Color online) The red, flatter surface shows the function $-\epsilon_{\text{total}}(\gamma, \theta)$. It results from computing and summing the ϵ -adroitness of experiments (a)–(d) in Fig. 1 when the system evolves according to the equation of motion (8). This surface divides the space into two regions: points above the surface do not violate $\mathcal{L} \geq -\epsilon_{\text{total}}(\gamma, \theta)$, while points below the surface do violate it. The blue, curvier surface shows the Leggett-Garg quantity \mathcal{L} as a function of the angle θ and the dephasing rate γ . The range of angles θ for which we observe a violation decreases as we increase the dephasing rate γ . At $\gamma > 0.007$, it is no longer possible to observe a violation for any angle θ . The less stringent condition $\mathcal{L} \geq 0$ allows a violation up to $\gamma > 0.012$ (not shown)

change equation (1) to $\mathcal{L} \geq -\epsilon_{\text{total}}$. This makes it harder to violate the Leggett-Garg inequality since only a substantially negative value of \mathcal{L} is unambiguous given our finite measurement adroitness. Figure 3 displays the Leggett-Garg quantity \mathcal{L} as a function of the angle θ and the dephasing rate γ . The range of angles θ for which we observe a violation of the Leggett-Garg inequality decreases as we increase the dephasing rate.

One can also study Fig. 2 as a function of dephasing rate γ . The violation exhibits a trade-off between the rate γ of dephasing noise and the number n of pairs of interleaved measurements because more measurements result in a longer amount of time for dephasing, and this extra dephasing then offsets the benefits of more measurements.

6 Conclusion

We have shown how to address a fundamental objection to the Leggett-Garg inequality by altering the protocol. We then explored improvements to our protocol and the effects of non-trivial system dynamics and noise. Our generic Hamiltonian and dephasing model could be replaced with more refined models, such as the spin-boson model [15], for calculations on specific systems.

Note that only strong, projective measurements appear in our protocol. Given the recent developments in weak-measurement-based Leggett-Garg tests [7, 8, 11–13], one might ask whether these tests have already addressed, or at least persuasively argued against, the clumsiness loophole. Unfortunately, the answer is no. The

weak measurement-based Leggett-Garg tests are derived using a quantitative non-invasiveness axiom [11] that assumes the detector noise $\xi(t)$ and the system variable $Q(t + \tau)$ are uncorrelated in time: $\langle \xi(t)Q(t + \tau) \rangle = 0$. The reason that a quantum system violates the weak measurement-based Leggett-Garg inequality is because it inevitably feels a sufficiently strong detector backaction $\langle \xi(t)Q(t + \tau) \rangle = f(\tau) \neq 0$. Since clumsy measurement of a macrorealistic system could also lead to strong detector backaction, a violation of the weak measurement-based Leggett-Garg inequality is perfectly consistent with the system being either (i) non-macrorealistic or (ii) macrorealistic but subjected to a measurement technique that happens to disturb the system so that the correlator $\langle \xi(t)Q(t + \tau) \rangle$ is non-zero. The clumsiness loophole has been rephrased but not mitigated at all.

Nor is the clumsiness loophole mitigated by interesting observations like: the weaker the measurement, the larger the violation of the inequality [7]. A macrorealist cannot even make sense of such a claim without a definition of measurement strength. The strength of a measurement is generally defined in terms of a quantum description of the system; a macrorealist does not accept the quantum description and would therefore require a definition of measurement strength in terms of some explicit experimental protocol. Such a protocol, which has not yet been framed, would likely lead to loopholes at least as large as our collusion loophole.

In conclusion, we have presented a revision of the Leggett-Garg protocol that improves the rigor of tests of macrorealism. This protocol should have applications both in quantum computing [16] and in the growing field of “quantum biology” [17, 18].

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