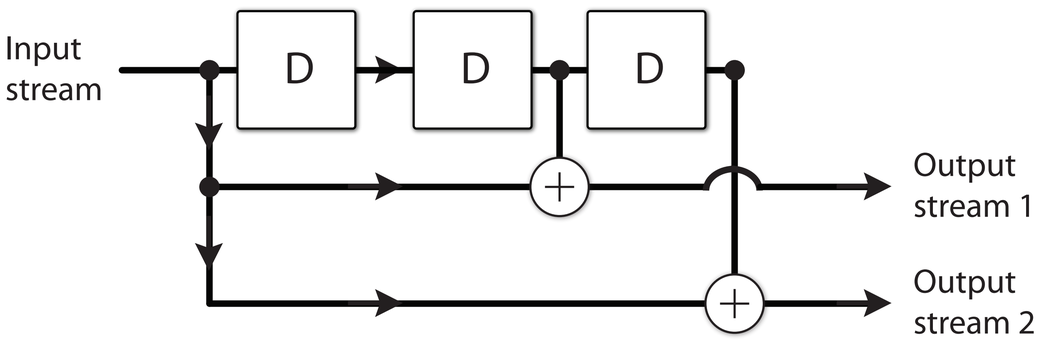
Quantum Shift-Register Circuits

Mark M. Wilde

School of Computer Science, McGill University,
Montreal, Quebec, Canada

Introduction

Classical shift register circuits
admit a simple form:



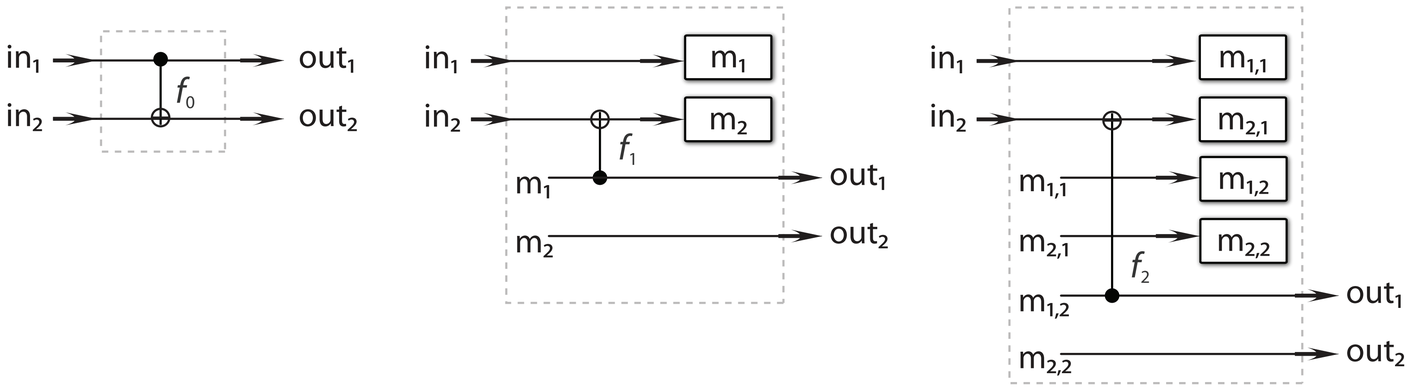
Does such a form exist for quantum shift register circuits?

Main Idea

A **quantum shift register circuit**
acts on a set of input qubits and memory qubits,
outputs a set of output qubits and updated memory qubits,
and feeds the memory back into the device for the next cycle
(similar to the operation of a classical shift register).

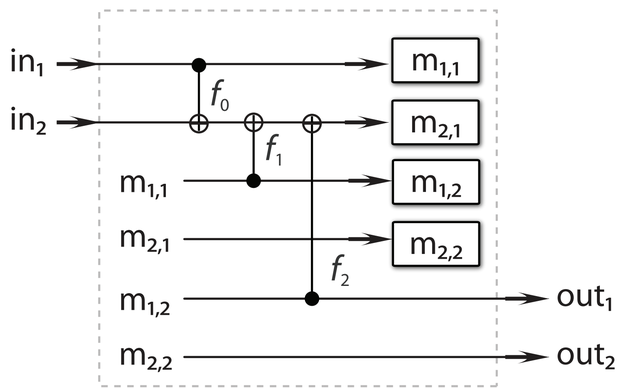
It is useful as an encoding or decoding circuit
in a **quantum convolutional code**.

Connect the Circuits



$$\left[\begin{array}{cc|cc} 1 & 0 & 0 & 0 \\ f_0 & 1 & 0 & 0 \\ 0 & 0 & 1 & f_0 \\ 0 & 0 & 0 & 1 \end{array}\right] \quad \left[\begin{array}{cc|cc} 1 & 0 & 0 & 0 \\ f_1 D^{-1} & 1 & 0 & 0 \\ 0 & 0 & 1 & f_1 D \\ 0 & 0 & 0 & 1 \end{array}\right] \quad \left[\begin{array}{cc|cc} 1 & 0 & 0 & 0 \\ f_2 D^{-2} & 1 & 0 & 0 \\ 0 & 0 & 1 & f_2 D^2 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

Makes



$$\left[\begin{array}{cc|cc} 1 & 0 & 0 & 0 \\ f(D^{-1}) & 1 & 0 & 0 \\ 0 & 0 & 1 & f(D) \\ 0 & 0 & 0 & 1 \end{array}\right]$$

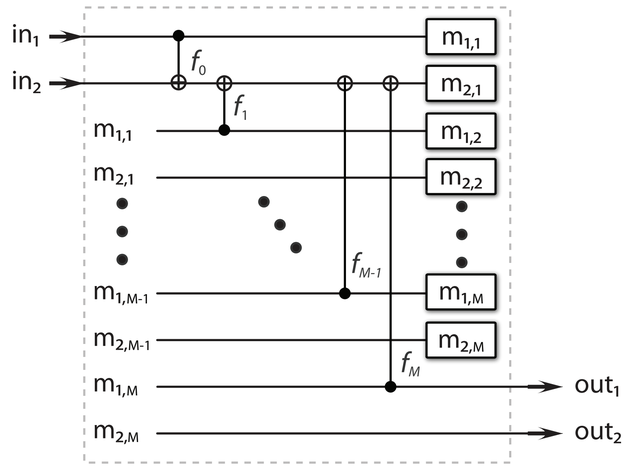
where

$$f(D) \equiv f_0 + f_1 D^1 + f_2 D^2$$

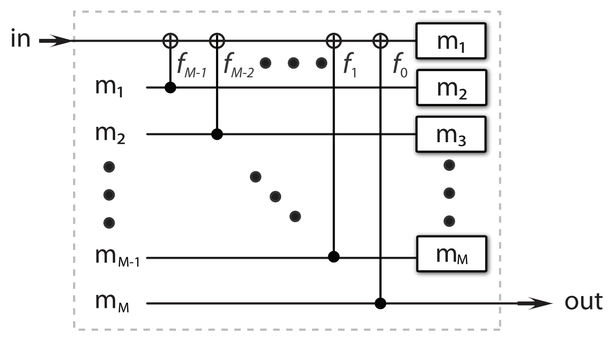
Building Blocks

Can build up encoding circuit
for **any** quantum convolutional code
with **building block shift register circuits**

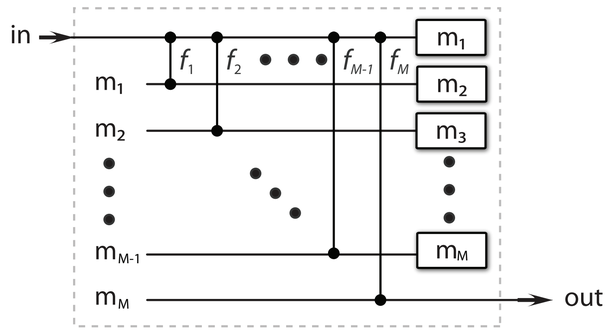
Finite-Depth CNOT



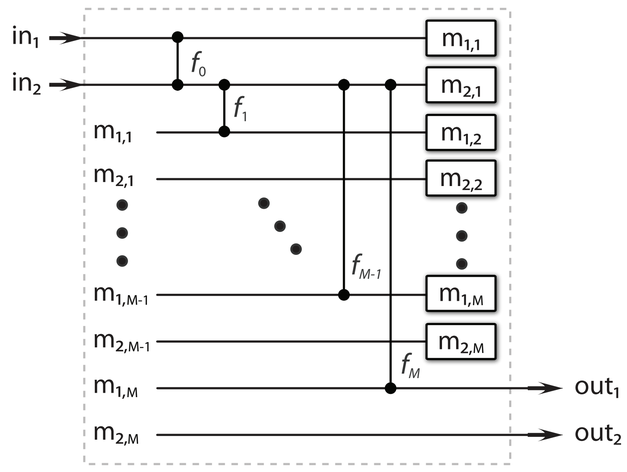
Infinite-Depth CNOT



Finite-Depth C-PHASE Type I



Finite-Depth C-PHASE Type II



Memory Use

Amount of memory required for CSS code
related to mathematical representation of code

Open problem for general codes...

M. M. Wilde. *Quantum-shift-register circuits*. Phys. Rev. A 79, 062325 (2009)